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Neutrino Properties

Neutrino physics is playing a unique role in elucidating the properties of weak interactions. But even more important, neutrino studies are capable of providing new ingredients for future theoretical descriptions of the physics of elementary particles. Although impressive progress has been made in our understanding of neutrino interactions, many important issues remain to be settled, foremost among these are the questions: have neutrinos a nonvanishing rest mass, and are neutrinos, participating in weak interaction, pure states in a quantum mechanical sense? The role of neutrino mass and its various consequences, therefore, will form the central issue throughout this text.

The study of neutrino properties, unlike the study of other elementary particles, has always crossed the traditional disciplinary boundaries. While we think of particle properties as being the domain of high energy physics, much information on neutrino properties has come from low energy nuclear physics, as well as from astrophysics and cosmology. For that reason it will be necessary to explore a number of different disciplines and techniques.

Among the many excellent reviews on weak interaction, we mention the books by Bailin (82), Commins & Bucksbaum (83), Georgi (84), Okun (82), Pietschmann (83), and Taylor (78). Since the first edition of this book several related books have appeared, Grotz & Klapdor (90), Holstein (89), and Kayser, Gibrat-Debu & Perrier (89). More specialized reviews on various aspects of neutrino physics are mentioned at the beginning of each Chapter.

This Chapter provides an introduction to the formal description of neutrinos and their properties. To begin, we briefly recapitulate in Section 1.1 the major milestones in the discovery of neutrinos, and outline the goal and purpose of this book. We review the fundamental properties of neutrinos with regards to charge conjugation in Section 1.2, elaborating on the important distinction between Dirac and Majorana neutrinos. In Section 1.3 we discuss the neutrino mass matrix and show how diagonalization of this matrix leads to the concept of neutrino oscillations. Finally, we explain in Section 1.4 how various fundamental particle theories may lead to finite neutrino mass.

1.1 Introduction

The history of neutrino physics began with Wolfgang Pauli's often quoted letter to the Physical Society of Tubingen (Pauli 30), in which he postulates the existence of a new particle, the neutrino, in order to explain the observed continuous electron spectrum accompanying nuclear beta decay. If, as it was then believed, the electron decay were a two-body decay, the laws of energy and momentum conservation would have predicted a monochromatic electron peak instead. Pauli required his hypothetical particle to be neutral and have spin 1/2, to ensure conservation of electric charge and angular momentum. Its rest mass was expected to be small, but not necessarily vanishing. Learning of Pauli's idea, Fermi (34) proposed his famous theory of beta decay, based on which Bethe & Peierls (34) predicted the cross section for the interaction of the neutrino with matter to be extremely small. The first experimental evidence of a neutrino induced interaction was brought by Reines & Cowan (53). In addition to the electron neutrino emitted in nuclear beta decay, two other neutrinos were discovered, the muon neutrino, demonstratively distinct from the electron neutrino (Danby et al. 62), and the the tau neutrino (Perl et al. 75), even though the latter's existence has only been inferred so far. The concept that these neutrinos could mix was proposed by Maki et al. (62) and by Pontecorvo (67). Earlier, Pontecorvo (58) discussed the possibility of neutrino-antineutrino oscillations.

As more and more experimental evidence became available, our understanding of weak interaction (and neutrino physics) was greatly revolutionized by the concept of parity-nonconservation (Lee & Yang 56). Experimental and theoretical developments led the way to modern weak interaction theories (Glashow 61; Salam 68; Weinberg 67) which became part of the more encompassing framework of the "standard model". This model is capable of describing all the known physics of weak and electromagnetic interactions, incorporating all the experimental results at energies available at present accelerators. The observation of the neutral weak current (Hasert et al. 73) and the discovery of the intermediate vector bosons W and Z (Arnison et al. 83; Bagnaia et al. 83) contributed spectacularly to the success of the model.

Yet, despite its success, the standard model appears in need of extension and generalization. In its present form it is not capable of predicting the masses of the fermions, nor can it explain why there are several fermion families (electron, muon, tau, and their neutrinos, and the analogous quark families). The study of neutrino properties is one of the few avenues which could lead to new physics beyond the standard model, and this is the chief reason why the neutrino is such an interesting particle. Earlier reports, indicating that neutrinos may have nonvanishing mass (see, e.g., Lubimov 86), although rejected by more recent work, have stimulated these endeavors.

One of the most important aspects of neutrino physics, therefore, has to do with its possible rest mass. We have no idea why neutrinos are so much lighter than the charged leptons, and we may only speculate that this difference may be a reflection of some fundamental symmetry. One possibility is that neutrinos, unlike any other fermions, are Majorana particles, that is they cannot be distinguished from their antiparticles.

If the neutrino is a massive Majorana particle, then neutrinoless double beta decay can take place. A search for this process is one of the most challenging tasks today. Also, if the neutrino has mass, then the question arises, is it a pure eigenstate of weak interaction, as described in the standard model, or is it a superposition of other neutrino states, the eigenstates of the mass matrix, as suggested in theoretical models of grand unification? If the neutrino is a mixed state in this sense, then a number of phenomena may take place, such as neutrino oscillations and neutrino decay.

Another aspect of neutrino physics has to do with the problem of lepton families (electrons, muons, and taus) and the related concept of lepton number conservation. With the help of neutrinos it is possible to test the lepton conservation laws to great accuracy.

These fundamental issues briefly sketched here can be addressed by experiment, and it is our aim to describe in this book both experiment and theory on an equal basis. From a theoretical point of view, the crucial issue is the question of whether the world of physics can be described by the standard model of electroweak interaction. If that were the case, the neutrino is predicted to be massless. The experiments on neutrino oscillations, neutrinoless double beta decay, neutrino decay, or searches for heavy neutrinos should give a null result, as they presently do at the limit of today's sensitivity. (In a few cases possible indications of effects associated with the neutrino mass have been reported, see, e.g., Section 4.1) If in the future one or several of the mentioned processes are unequivocally found, it would point the way to some more general theory.

1.2 Dirac and Majorana Neutrinos

It was Dirac's equation that first led to the concept of particles and antiparticles, the positive electron being the earliest candidate for an antiparticle. While positive electrons are clearly distinct from negative electrons by their electromagnetic properties, it is not obvious in what way neutral particles should differ from their antiparticles. The neutral pion, for example, was found to be identical to its antiparticle. The neutral kaon, on the other hand, is clearly different from its antiparticle. The pion and kaon, both bosons, are not truly elementary particles, however, as they are composed of two charged fermions, the quarks and the antiquarks.

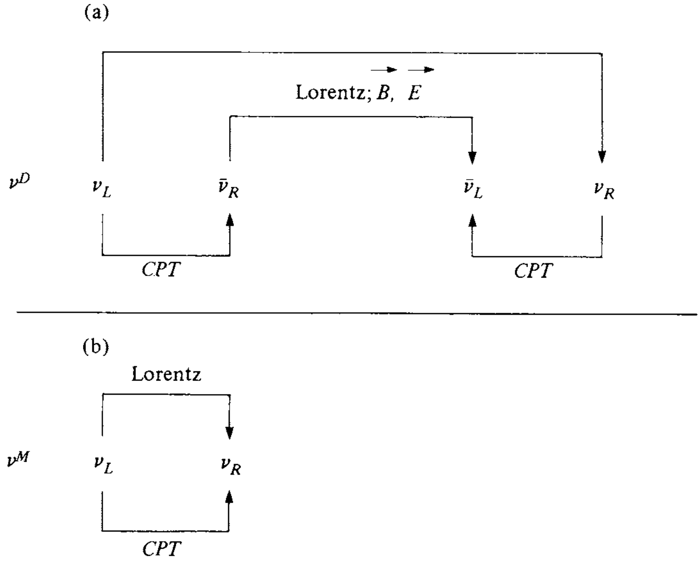
The concept of a particle which is identical to its antiparticle was formally introduced by Majorana as early as 1937 (Majorana 37). Thus, we refer to such a particle as a Majorana particle. In contrast, a Dirac particle is one which is distinct from its antiparticle. In this Section we shall describe the properties of Majorana particles in general, and neutrinos in particular, and how they can be distinguished from Dirac particles.

The difference between a Majorana and Dirac particle has to do with its transformation properties under charge conjugation. However, as far as we know, neutrinos interact only weakly, and weak interactions are not invariant with respect to charge conjugation. Consequently, an interacting Majorana neutrino cannot be an eigenstate of charge conjugation C alone. (If a Majorana neutrino had a definite value of C at one instant, it would no longer have this value at a later time, because weak interaction would mix in other eigenstates of C.) Therefore, we have to generalize the definition of a Majorana particle and also consider transformation properties with respect to other discrete symmetries, such as parity P and time reversal T (or their combinations CP and CPT). Moreover, weak interactions involve neutrino states of a selected chirality (left-handed or right-handed) and thus the formalism should be based on the chiral projections of the corresponding states.

To visualize the difference between Majorana and Dirac neutrinos, let us follow the arguments of Kayser (85) and assume the existence of a massive left-handed neutrino ν_L (see Figure 1.1). (We shall show later that there is an intimate relation between helicity, i.e., handedness, and chirality, i.e., the transformation property with respect to the application of γ_5.) By virtue of CPT invariance which we assume to be valid, the existence of ν_L implies the existence of the CPT mirror image, a right-handed antineutrino $\bar{\nu}_R$. As a massive ν_L travels slower than light, a frame of reference moving faster than ν_L exists. In this frame the neutrino is going the other way, but its spin is unchanged. Thus, the Lorentz transformation to the faster moving frame of reference turns ν_L into a right-handed ν_R, shown at the extreme left of Figure 1.1(a). Now, this ν_R may or may not be the same particle as the CPT mirror image $\bar{\nu}_R$ of ν_L. If it is *not* the same, as indicated in Figure 1.1(a), ν_R has its own CPT mirror image $\bar{\nu}_L$. Altogether, there are four states with the same mass. This quadruplet of states is called a Dirac neutrino ν^D. It has distinct particle and antiparticle states, may have a magnetic dipole moment, and, if CP is not conserved, even an electric dipole moment. Thus, for a Dirac neutrino, ν_L can be converted into the opposite helicity state both by a Lorentz transformation and by the torque exerted by an external $\vec{B}$ or $\vec{E}$ field.

If, on the other hand, the right-handed particle obtained by the Lorentz transformation to a moving reference frame is the *same* particle as the CPT image of the original ν_L (Figure 1.1(b)), there are only two states with a common mass. This pair of states represents a Majorana neutrino ν^M. It is easy to see that the dipole magnetic and electric moments of ν^M must vanish. Indeed, at rest in an external static field the neutrino would have the interaction energy $-\mu<\vec{s}\cdot\vec{B}>-d<\vec{s}\cdot\vec{E}>$, where $\vec{s}$ is the neutrino spin operator and μ and d are the corresponding dipole moments. Under CPT the fields $\vec{E}$ and $\vec{B}$ are unchanged, while the spin vector of a Majorana neutrino reverses its direction. Therefore, under CPT the dipole interaction

Figure 1.1. (a) The four distinct states of a Dirac neutrino ν^D. (b) The two distinct states of a Majorana neutrino ν^M.



energy changes sign. Consequently, if *CPT* invariance holds, the dipole moments μ and d must vanish.

It should be stressed that the existence of the neutrino rest mass was essential for the above discussion of the differences between Dirac and Majorana neutrinos. A massless neutrino travels with the speed of light and we can no longer reverse its helicity by going to the faster moving Lorentz frame. Moreover, let us assume that the weak interaction involves only left-handed currents. (This agrees with experimental evidence, interactions involving right-handed currents have not been detected so far.) For neutrinos with left-handed interaction, the dipole moment is proportional to mass and vanishes for a massless particle. Thus, the helicity of a massless neutrino cannot be reversed by an external $\vec{E}$ or $\vec{B}$ field, either. Indeed, in the massless case the states $|\nu_L^D>$ and $|\bar{\nu}_R^D>$ are completely disconnected from the remaining two states $|\nu_R^D>$ and $|\bar{\nu}_L^D>$; the latter states need not even exist. As there are only two relevant states, the Dirac-Majorana distinction has vanished. Actually, the disappearance of the distinction between the Dirac and Majorana neutrinos is not abrupt. As the mass gets smaller (or as the mass/energy gets smaller), the ability to decide whether the observed neutrino states are the two spin states of a Majorana neutrino or half of the four states of a Dirac neutrino, gradually vanishes.

1.2.1 *Definitions and effect of discrete symmetries*

Throughout this Chapter we shall use the conventions and notation of Sakurai (64). Unless noted otherwise, we shall work with the Dirac-Pauli representation of the gamma matrices (with a change of sign of γ_5 to make it compatible with other conventions):

$$\vec{\gamma} = \begin{pmatrix} 0 & -i\vec{\sigma} \\ i\vec{\sigma} & 0 \end{pmatrix} ; \; \gamma_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} ; \; \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} , \tag{1.1}$$

where σ are Pauli matrices.

A free neutrino field ψ_i is a four-component object which obeys the Dirac equation

$$\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + m_i \right) \psi_i = 0 \, . \tag{1.2}$$

The field $\gamma_5\psi_i$ satisfies an equation in which the sign of mass is reversed,

$$\left(\gamma_\mu \frac{\partial}{\partial x_\mu} - m_i \right) \gamma_5\psi_i = 0 \, . \tag{1.3}$$

The fields with a definite chirality (eigenstates of γ_5) are the projections,

$\frac{(1 \pm \gamma_5)}{2}\psi$. It is obvious from (1.2) and (1.3) that a free massive particle cannot have a definite chirality at all times. On the other hand, in the ultrarelativistic limit $E/m \rightarrow \infty$ (or for massless particles) the chirality becomes a good quantum number. There is an intimate relation between the chirality and helicity (i.e., spin projection on the direction of motion). The helicity operator is $\vec{\Sigma}\cdot\hat{p}$, where $\vec{\Sigma} = -i\gamma_4\gamma_5\vec{\gamma}$. Then $\gamma_5 \rightarrow \vec{\Sigma}\cdot\hat{p}$ in the ultrarelativistic limit for positive energy states. We shall follow the often used, but not very precise custom, and refer to the corresponding chiral projections $\frac{(1 \pm \gamma_5)}{2}\psi$ as right- and left-handed, respectively.

The charge conjugate field ψ_i^c is defined as

$$\psi_i^c = \eta_C C \overline{\psi}^{\,T} , \tag{1.4}$$

where η_C is a phase factor, C is a 4×4 matrix, (the symbol C is customarily used for two meanings, as the symbol of the charge conjugation operation and as the 4×4 matrix) and the superscript T signifies a transposed matrix (as usual $\overline{\psi} = \psi^+\gamma_4$). In the Dirac-Pauli representation $C = \gamma_4\gamma_2$ so that $\psi^c = -\eta_C\gamma_2\psi^*$. Note that the following relations are independent of representation:

$$C^{-1}\gamma_\mu C = -\gamma_\mu^T \, ; \;\; C^{-1}\gamma_5 C = \gamma_5^T \, .$$

If ψ is a chirality eigenstate ($\gamma_5\psi = \lambda\psi$), then ψ^c is also a chirality eigenstate with eigenvalue $\lambda^c = -\lambda$.

Next, we shall consider the effect of the parity transformation ($\vec{x}' = -\vec{x}$, $x_4' = x_4$) on the field ψ. Requiring that the Dirac equation in both systems describes the same situation leads to the rule

$$\psi(\vec{x}) \rightarrow \eta_P\gamma_4\psi(-\vec{x}) , \tag{1.5}$$

where η_P is a phase factor. The charge conjugate field ψ^c transforms under the parity transformation as

$$\psi^c \rightarrow \eta_C\eta_P^* C((\gamma_4\psi)^+\gamma_4)^T = -\eta_P^*\gamma_4\psi^c \, . \tag{1.6}$$

Thus, for real η_P, we obtain the well-known result that the intrinsic parity of the antifermion is opposite to that of a fermion. (This can be experimentally verified, for example, by a study of the positronium decay.) If, however, ψ should describe a Majorana particle (charge conjugation eigenstate), one can satisfy Eq. (1.6) only with pure imaginary η_P. Hence we come to important conclusion: *Majorana particles have imaginary intrinsic parity.*

The Majorana field can be defined as

$$\chi(x) = \frac{1}{\sqrt{2}} \, [\psi(x) + \eta_C\psi^c(x)] \, . \tag{1.7}$$

By the appropriate choice of phase (if $\eta_C = e^{2i\Phi}\lambda_C$, where $\lambda_C = \pm 1$, choosing $\chi' = e^{-i\Phi}\chi$) we obtain a field which is an eigenstate of charge conjugation with the eigenvalue $\lambda_C = \pm 1$. The Majorana field (1.7) has the CP phase $\pm i$; fields with $+i$ will be called even CP states, and those with $-i$ will be called odd CP states. (Until now we have considered only free fields. Once interactions, for example weak interactions of the neutrinos, are included, the requirement of C invariance alone is insufficient, as we mentioned above. Thus, we have to require that the Majorana field is an eigenstate of CPT, or at least of CP. For our purpose, however, it is sufficient to use the definition (1.7).) If the Majorana state χ, or state $\chi' = e^{-i\Phi}\chi$, has the charge conjugation eigenvalue λ_C, the state $\gamma_5\chi'$ has the opposite eigenvalue $-\lambda_C$.

As mentioned, the chiral projections, $\frac{(1 \pm \gamma_5)}{2}\psi$, do not obey the Dirac equation unless $m=0$. We use the following notation for the chiral projections of the fields ψ and the charge conjugate field ψ^c

$$\psi_L \equiv \frac{(1-\gamma_5)}{2}\psi \; ; \; (\psi_L)^c = \frac{(1+\gamma_5)}{2}\psi^c = (\psi^c)_R \, , \tag{1.8a}$$

$$\psi_R \equiv \frac{(1+\gamma_5)}{2}\psi \; ; \; (\psi_R)^c = \frac{(1-\gamma_5)}{2}\psi^c = (\psi^c)_L \, . \tag{1.8b}$$

It is obvious that the charge conjugation eigenstates (1.7) cannot simultaneously be eigenstates of chirality.

1.2.2 Mass term for a single field

In the field theory of neutrinos the mass is determined by the mass term of the neutrino Lagrangian. This mass term must be Lorentz invariant and hermitian. This requirement restricts the possible mass terms to two groups: $\bar{\psi}\psi$ and $\bar{\psi^c}\psi^c$, as well as $\bar{\psi}\psi^c$ and its hermitian conjugate $\bar{\psi^c}\psi$. The Lagrangian of a single Dirac field has the form

$$\mathbf{L}_D = -\frac{1}{2}\int d^4x \, (\bar{\psi}\gamma_\mu\partial_\mu\psi + \bar{\psi} m_D \psi) \, , \tag{1.9}$$

and we see that it contains the mass term of the first kind.* (The parameter m_D is real to preserve hermicity of the Lagrangian.) This Dirac mass term is invariant under the global phase transformation

$$\psi \rightarrow e^{i\alpha}\psi \; ; \; \psi^c \rightarrow e^{-i\alpha}\psi^c \, .$$

The second possible Lorentz invariant mass term containing $\bar{\psi}\psi^c$ and $\bar{\psi^c}\psi$ is

* The first part of (1.9) is the kinetic energy term. We shall not discuss it further, but one has to make sure that any transformation of the mass part leaves the kinetic energy part invariant.

not invariant under this phase transformation. Thus, the Dirac mass term can be associated with a conserved quantum number (usually called the "lepton number") while the second Lorentz invariant mass term violates conservation of the lepton number by two units. It will become apparent shortly why this second mass term is known as the Majorana mass term.

Let us consider the most general mass Lagrangian

$$-2\mathrm{L}_M = \frac{1}{2}\left(\bar{\psi} m_D \psi + \bar{\psi}^c m_D \psi^c + \bar{\psi} m_M \psi^c + \bar{\psi}^c m_M^* \psi \right) . \qquad (1.10)$$

This Lagrangian is hermitian and Lorentz invariant; it depends on three real parameters m_D, m_1, m_2 ($m_M = m_1 + im_2$). It is instructive to rewrite this Lagrangian in the matrix form

$$-2\mathrm{L}_M = \frac{1}{2}\begin{pmatrix}\bar{\psi}, & \bar{\psi}^c\end{pmatrix}\begin{pmatrix} m_D & m_M \\ m_M^* & m_D\end{pmatrix}\begin{pmatrix}\psi \\ \psi^c\end{pmatrix} . \qquad (1.10a)$$

To find fields of a definite mass, we have to diagonalize (1.10a). When this is done, we find that the mass term (1.10a) has two real eigenvalues $m_D \pm |m_M|$, and the corresponding eigenvectors are

$$\begin{pmatrix}\varphi_+ \\ \varphi_-\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta}\psi + e^{i\theta}\psi^c \\ -e^{-i\theta}\psi + e^{i\theta}\psi^c\end{pmatrix} , \qquad (1.11)$$

where $\tan 2\theta = m_2/m_1$. Both eigenvectors are Majorana fields, i.e., they are eigenstates of charge conjugation with opposite eigenvalues. (If $m_M = 0$ in (1.10), the mass term is already diagonal with a single eigenvalue m_D; the eigenvectors ψ and ψ^c in that case are not charge conjugation eigenstates and describe a Dirac particle.)

There is a difficulty associated with the diagonalization of (1.10). This mass term, being hermitian, has real eigenvalues. But these eigenvalues are not always positive, while it is customary to consider the particle rest mass a positive quantity. As long as $m_D > |m_M|$, this does not represent a problem, because both mass eigenvalues are positive. If, on the other hand, $|m_M| > m_D$, one of the eigenvalues is negative. When that happens, we can use the γ_5 trick (see (1.3)) and change the sign of m by using the field $\gamma_5\varphi_-$ instead of φ_-. In that case, however, both solutions have a positive C eigenvalue. In the special case of pure Majorana mass ($m_D = 0$), the two mass eigenvalues are degenerate and we can use any combination of φ_+ and $\gamma_5\varphi_-$ as the appropriate eigenvector.

If, instead, we work with chiral projections of the field ψ, we can use the fact that $\bar{\psi}_L\varphi_L = \bar{\psi}_R\varphi_R = 0$ for any fields ψ and φ (since $(1+\gamma_5)(1-\gamma_5) = 0$). Thus, the Dirac mass term, expressed in terms of chiral projections is either $\bar{\psi}_L\psi_R$ or $\bar{\psi}_R\psi_L$, while the Majorana mass term has the form $\bar{\psi}_L(\psi^c)_R$, etc.

The mass terms of both kinds "violate chirality" because, as we have remarked already, chirality is not a conserved quantity for massive particles.

When the mass term is expressed in terms of the chiral projections, the number of parameters, and the dimension of the mass matrix, doubles. The most general hermitian mass term, written in a form analogous to (1.10a) is now

$$-2L_M = \frac{1}{2}\left(\bar{\psi}_R, (\bar{\psi^c})_R, \bar{\psi}_L, (\bar{\psi^c})_L\right)\begin{pmatrix} 0 & M \\ M^+ & 0 \end{pmatrix}\begin{pmatrix} \psi_R \\ (\psi^c)_R \\ \psi_L \\ (\psi^c)_L \end{pmatrix}, \qquad (1.12)$$

where the 2×2 submatrix M depends on three complex numbers, and can be expanded in terms of the Pauli matrices

$$M \equiv m_D I + m_1\sigma_x + m_2\sigma_y \, . \qquad (1.13)$$

The mass term (1.12) is automatically *CPT* invariant. If, further, *CP* invariance is required, the parameters m_D and m_1 must be real and m_2 must be pure imaginary.

Diagonalization of the mass term (1.12) is relatively simple in the *CP* invariant situation. (The treatment of the general *CP* noninvariant case is discussed by Rosen (83); throughout this Section we use many results from that reference.) The eigenvalue equation is of the form

$$\lambda_{\pm} = \frac{1}{2}\left\{(m_R + m_L) \pm [(m_R - m_L)^2 + 4m_D^2]^{\frac{1}{2}}\right\}, \qquad (1.14)$$

where

$$m_R = m_1 + |m_2| \, , \; m_L = m_1 - |m_2|$$

for $m_2 = +i|m_2|$. (For $m_2 = -i|m_2|$ we must interchange m_L and m_R.) The eigenvectors, as in the nonchiral case, are C or *CP* eigenstates (but, naturally, not chirality eigenstates). Again, we may encounter negative mass eigenvalues, which could be removed by the "γ_5 trick". Eq. (1.12) is formally four-dimensional; in reality, eigenvalues and eigenvectors appear in pairs and only two are independent.

1.2.3 Concluding remarks

We have shown that the general mass term (1.10) or (1.12) has eigenvectors which are charge conjugation eigenstates, i.e., they describe Majorana particles. Typically, these states are also *CP* eigenstates. The