

This book presents a systematic treatment of a broad area of modern optical physics dealing with coherence and fluctuations of light. It is a field that has largely developed since the first lasers became available in the 1960s.

The first three chapters cover various mathematical techniques that are needed later. A systematic account is then presented of optical coherence theory within the framework of classical optics, and this is applied to subjects that have not been treated systematically before, such as radiation from sources of different states of coherence, foundations of radiometry, effects of source coherence on the spectra of radiated fields, coherence theory of laser modes and scattering of partially coherent light by random media.

A semiclassical description of photoelectron detection precedes the treatment of field quantization and of the coherent states, and this is followed by a discussion of photon statistics, the quantum theory of photoelectric detection and applications to thermal light. This includes a discussion of correlation measurements and photon antibunching, and of the Einstein–Podolsky–Rosen locality paradox.

A chapter is devoted to the interaction between light and a two-level atom and the problem of resonance fluorescence, and this is followed by treatments of cooperative radiation effects. After describing some general techniques for analyzing interacting quantum systems, such as the regression theorem and master equations, the book goes on to treat the single-mode and the two-mode laser and the linear amplifier. The concluding chapters deal with squeezed states of light and their generation and detection, and with some quantum effects in nonlinear optics such as parametric down-conversion, phase conjugation and quantum non-demolition measurements. Each chapter concludes with a set of problems.

The authors are well-known scientists who have made substantial contributions to many of the topics treated in this book. Much of the book is based on various graduate courses given by them over the years.

This book is likely to become an indispensable aid to scientists and engineers concerned with new developments in modern optics, as well as to teachers and graduate students of physics and engineering.

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# Optical coherence and quantum optics

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*Dedicated to our wives*  
*Jeanne and Marlies*  
*in appreciation of their patience, understanding and help*

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## Preface

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Prior to the development of the first lasers in the 1960s, optical coherence was not a subject with which many scientists had much acquaintance, even though early contributions to the field were made by several distinguished physicists, including Max von Laue, Erwin Schrödinger and Frits Zernike. However, the situation changed once it was realized that the remarkable properties of laser light depended on its coherence. An earlier development that also triggered interest in optical coherence was a series of important experiments by Hanbury Brown and Twiss in the 1950s, showing that correlations between the fluctuations of mutually coherent beams of thermal light could be measured by photoelectric correlation and two-photon coincidence counting experiments. The interpretation of these experiments was, however, surrounded by controversy, which emphasized the need for understanding the coherence properties of light and their effect on the interaction between light and matter.

Undoubtedly it was the realization that the subject of optical coherence was not well understood that prompted the late Dr E. U. Condon to invite us, more than three decades ago, to prepare a review article on the subject of coherence and fluctuations of light for publication in the *Reviews of Modern Physics*, which he then edited. The article was well received and frequently cited, and this encouraged us to expand it into a book. Little did we know then how rapidly the subject would develop and that it would become the cornerstone of an essentially new field, now known as quantum optics. Also the first experiments dealing with non-classical states of light were reported in the 1970s, and they provided the impetus for the new quantum mechanical developments. As an indication of the growth of the field we note that the book *Principles of Optics*, by M. Born and E. Wolf, published in 1959, the year before the laser was invented, had a chapter of just over 60 pages on partially coherent light, which covered most of what was then known about the subject. It was based entirely on the classical wave theory; quantum optics barely existed at that time. By contrast, in the present book more than twice as much space is devoted to quantum as to classical phenomena. The book is perhaps unusual in covering both the classical and the quantum theory of fluctuating electromagnetic fields in some depth.

Despite the length of the book, we make no claim as to its completeness, especially with respect to the quantum mechanical sections, and several topics are treated only cursorily or not at all. For example, only a short section deals with the subject of laser cooling and trapping, which has grown to merit a book of its own, and the important new field of atom interferometry is not treated at all.

Although at first we tried to be consistent in the use of notation throughout the book, later we abandoned the attempt, in part because the size of the book made it impractical, and partly because the use of certain symbols has become standard in some subfields. As regards the much debated question of the best choice of units for electromagnetic quantities, we have demonstrated our open-mindedness by employing both Gaussian and SI units. However, SI units are always used in discussing experiments.

Much of the book is an outgrowth of lectures that we have both given over more than 30 years at the University of Rochester, New York, and elsewhere. In particular Section 3.2 of the book on the angular spectrum representation of wavefields is based on lectures first given by one of us (EW) at the former National Bureau of Standards in Gaithersburg, Maryland in 1979 and 1980. Part of the text was prepared by him during sabbatical leaves at the University of California in Berkeley and at the Schlumberger–Doll Research Laboratory in Ridgefield, Connecticut, and he wishes to acknowledge his indebtedness to Professor Sumner P. Davis and to Dr Robert P. Porter for providing congenial facilities for the work.

We wish to thank Mr K. J. Harper and Mrs P. T. Sulouff, the former and the present Head Librarians of the Physics, Optics and Astronomy library at the University of Rochester, for their assistance in tracing and checking some of the less accessible references.

Authors from academia are often fortunate in being able to call on their present and former graduate students for assistance, and we gratefully acknowledge the help we received from several generations of our students in checking various sections of the manuscript and making many valuable suggestions for improvement. We are particularly indebted to G. S. Agarwal, S. Bali, D. Branning, B. Cairns, F. C. Cheng, D. G. Fischer, A. Fougères, A. Gamliel, T. P. Grayson, D. F. V. James, M. Kowarz, P. D. Lett, F. A. Narducci, J. W. Noh, J. R. Torgerson, L. J. Wang, W. Wang, and X. Y. Zou for their help. We are also grateful to Mr Fischer for preparing the subject index and to Dr W. Wang for having drawn some of the figures and for checking many of the references.

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