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B. BOLLOBAS, P. SARNAK, C. T. C. WALL

103 Designs and their codes

E. F. ASSMUS, JR.
Department of Mathematics, Lehigh University

J. D. KEY
Department of Mathematical Sciences, Clemson University

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Preface

The aim of this book is to study applications of algebraic coding theory to the analysis and classification of designs. Designs are usually classified by their parameter sets, by their inclusion in infinite families, or according to the type of automorphism group they admit — or in other ways related to their geometric properties. Here we have tried to confront the following questions. To what extent can algebraic coding theory address classification questions concerning designs? Can it assist in their construction? Does it provide insight into the structure and nature of classes of designs? We have tried to outline ways in which these questions have been answered and to point to areas where coding theory can make a valuable contribution to design theory.

These matters occupy the last three chapters of the book; the first five chapters are of a background nature and form an introduction to both the theory of designs and algebraic coding theory. Although parts of these chapters are elementary we hope they will be of interest even to the expert. We have rethought much of the material and some of the development is new. We have not always given complete proofs and in several instances we have given no proof at all. On the other hand there are occasions where we have given more than one proof for the same result. Our guiding principle has been to use the proofs as didactic aids rather than verifications of the assertions; in particular, where the proof in the literature is clear and easily available but would add little to the exposition, we have omitted it.

We have included a glossary of terms and symbols that we hope will aid the expert who wishes to jump into one of the last three chapters without consulting the first five. Thus, for example, someone interested only in what coding theory might have to say about Hadamard matrices will be able to begin by reading Chapter 7 — using the glossary and index in the event that a concept or the notation is unfamiliar. Similarly, someone interested in projective planes — and their coding-theoretic habitat — ought to be able to go directly to Chapter 6 and the Steiner-system expert directly to Chapter 8.

The numbering is standard and is consecutive within sections: thus the third exercise of the second section of the first chapter is Exercise 1.2.3. The theorems, propositions, corollaries, lemmas and examples are similarly listed. In the **Bibliography** the abbreviations for the titles of journals are those used by *Mathematical Reviews*; the full journal title is given in the event *Mathematical Reviews* does not list an abbreviation or the abbreviation might prove confusing. Volume numbers are in Arabic numerals in keeping with current library custom. We have tried to be accurate in reporting the historical developments and assigning credit; moreover, we have cited all the works that we have consulted. We apologize in advance for any omissions and slights that perhaps remain.

The book was begun on 11 February 1990; the construction of the "manuscript" and the collaborative effort was almost exclusively electronic, using the EMACS editor in conjuction with LaTeX and relying on NSFNET to transfer files between us. We have, for this second printing, updated the bibliography and corrected all the typographical errors that have come to our attention since publication, but otherwise the book is essentially the same as the original.

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