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978-0-521-41269-8 - Paradoxes of Belief and Strategic Rationality

Robert C. Koons

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This book develops a framework for analyzing *strategic rationality*, a notion central to contemporary game theory, which is the formal study of the interaction of rational agents and which has proved extremely fruitful in economics, political theory, and business management.

The author argues that a logical paradox (known since antiquity as the “Liar paradox”) lies at the root of a number of persistent puzzles in game theory, in particular those concerning rational agents who seek to establish some kind of reputation. Building on the work of Parsons, Burge, Gaifman, and Barwise and Etchemendy, Robert Koons constructs a context-sensitive solution to the whole family of liar-like paradoxes, including, for the first time, a detailed account of how the interpretation of paradoxical statements is fixed by context. This analysis provides a new understanding of how the rational agent model can account for the emergence of rules, practices, and institutions.

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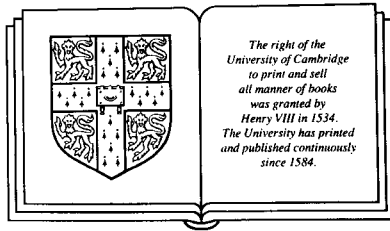
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*Paradoxes of belief
and
strategic rationality*

Robert C. Koons

University of Texas at Austin



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To Debbie, my wife and dearest friend

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Preface

My principal aims in writing this book are two. First of all, I am attempting a defense of a thesis that I call “Computationalism,” namely, the thesis that the objects of psychological attitudes (like belief and intention) are abstract objects which are structured in a way that mirrors the syntactic structure of sentences. Moreover, I intend to defend a version of computationalism that is relatively “type-free” in the sense that it permits these objects of belief and other attitudes to be genuinely self-referential or self-involving. I will defend this thesis against one particular objection: the claim that such a type-free, computationalist approach to the psychological attitudes is untenable because it leads to theories afflicted by versions of the liar paradox. My defense against this objection will be twofold: I will argue that the abandonment of computationalism is neither sufficient nor necessary for averting liar-like paradoxes.

Second, I will make a case for thinking that an understanding of liar-like paradoxes is crucial to those branches of social science (like economics, game theory, public-choice and social-contract political theory, organization theory, and Gricean linguistics) that employ some form of the *rational agent model*. A notion that plays an important role in many applications of this model is that of common knowledge, or *mutual belief*. I claim that a liar-like paradox will emerge in any adequate theory of this notion that is sensitive to issues of computational complexity. Moreover, I argue that solving certain puzzles involving the notion of *reputation* depends on recognizing the role that liar-like paradoxes play in generating these puzzles. Finally, I conclude by speculating that liar-like paradoxes will play a crucial role in effecting the transition from the rational agent model to an understanding of society in terms of institutions, rules, and practices.

I argue in Chapter 1 that doxic paradoxes (liar-like paradoxes involving the notion of *rational belief*) can be formulated without relying on objects of belief that are self-referential or self-involving.

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Even in an intensional logic using only modality-like operators (which identify the objects of belief with sets of possible worlds), it is possible to show that very plausible principles of rational belief come into conflict. This indicates that avoiding the liar-like paradoxes that afflict type-free computational theories of belief is not a sufficient motive for rejecting computationalism, since paradoxes afflict noncomputationalist accounts as well.

In Chapter 2, I show that the doxic paradoxes isolated in Chapter 1 are at the root of a family of puzzles in contemporary game theory. These puzzles concern the possibility of building or sustaining a tit-for-tat reputation in finitely repeated games, such as Selten's chain-store paradox. I criticize several recent attempts to solve these puzzles, including Sorensen's theory of "blindspots."

In Chapter 3, I argue that a paradox-generating epistemic logic (the logic of knowledge) can be extracted from the practice of contemporary metamathematics (the mathematical study of proofs and other formal methods in mathematics itself). Using the techniques of standard modal logic, I show that the liar paradox, the paradox of the knower, and a variety of related doxic paradoxes can be seen as special cases of a general phenomenon.

In Chapter 4, I develop a representational, computational account of the phenomenon of common belief, a notion that plays a crucial role in game theory and in linguistics. I demonstrate that an adequate theory of mutual belief that takes computational limitations into account will be afflicted with the sort of doxic paradoxes isolated in Chapters 1 through 3.

Part II takes up the question of solutions to these paradoxes. I criticize several context-insensitive solutions to the liar paradox in Chapter 5, including truth-value gap theories, oscillating extensions (Gupta and Herzberger), and recent work by Terence Parsons and Solomon Feferman. In Chapter 6, I compare three context-sensitive theories: those of Tyler Burge, Jon Barwise and John Etchemendy, and Haim Gaifman. I demonstrate that all three are special cases of a more general theory, and I propose certain specific revisions in Gaifman's algorithm for assigning indices to tokens. Finally, in Chapter 7, I adapt this algorithm to the case of the doxic paradoxes.

I presuppose some knowledge of first-order logic and elementary probability theory throughout the book. Very little else is needed for reading most of it. Although Chapter 2 includes some discussion of

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recent game-theoretic literature, I have tried to make it accessible to the non-game-theorist. Chapter 3 reviews some technical results in metamathematics and contains some comparisons with modal logic but should not require any special expertise in either area. Chapter 5 should probably be skipped by someone who is unfamiliar with recent work on the liar paradox. Chapters 6 and 7 contain several rather technical definitions that can be skipped, but both also contain some quite informal discussion of the nature and relevance of the context-sensitive solutions to the paradoxes.

Chapter 1 appeared as “Doxastic Paradox without Self-Reference” in the *Australasian Journal of Philosophy* 68 (1990): 168–77. Chapter 2 appeared as “A Representational Account of Mutual Belief” in *Synthese* 81 (1989): 21–45. A version of Chapter 6 appeared as “Three Indexical Solutions to the Liar Paradox” in *Situation Theory and Its Applications*, Vol. 1, edited by Robin Cooper, Kuniaki Mukai, and John Perry (Center for the Study of Language and Information, Stanford, Calif., 1990); Part of Chapter 7 appeared as “Doxic Paradox: A Situational Approach” in *Situation Theory and Its Applications*, Vol. 3, edited by J. Barwise, J. M. Gawron, G. Plotkin, and S. Tutiya (CSLI, Stanford, Calif., 1991). I thank the editors of these journals and CSLI for permission to reprint this material here.

I thank my teacher, Tyler Burge, who supervised and guided much of the research appearing here. I also thank David Charles and Robert M. Adams, both of whom helped greatly when my ideas were in the seminal stage. In addition, I thank my colleagues at Texas, especially Dan Bonevac, Nicholas Asher, and Bob Causey, who provided indispensable feedback and suggestions. I especially thank my editor, Brian Skyrms, for all of his help and encouragement, as well as his very valuable criticisms and suggestions. Much of the work was completed with the support of a National Science Foundation grant to the Center for Cognitive Science at the University of Texas (IRI-8719064) and a Summer Research Assignment from the University Research Institute of the University of Texas at Austin. Their help is gratefully acknowledged. Finally, I thank my parents, Bruce and Margaret Koons, for all their support and faith in me over many years, and my wife Debbie, without whom nothing I do would be possible.