Cambridge University Press
978-0-521-41152-3 - Laminar Flow Analysis
David F. Rogers
Frontmatter
More information

Laminar Flow Analysis

LAMINAR FLOW ANALYSIS

David F. Rogers

Professor of Aerospace Engineering
United States Naval Academy
Annapolis, Maryland

CAMBRIDGE
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, Victoria 3166, Australia

First published 1992

Printed in the United States of America

Library of Congress Cataloging-in-Publication Data
Rogers, David F., 1937-
Laminar Flow Analysis / David F. Rogers.
p. cm.
Includes bibliographical references.
ISBN 0-521-41152-1
1. Laminar flow. 2. Boundary layer. 3. Navier-Stokes equations.
I. Title.
TL574.L3R64 1992
629.132'32—dc20 92-9137
CIP

A catalog record for this book is available from the British Library.

ISBN 0-521-41152-1 hardback

Dedicated to
Professor Henry T. Nagamatsu
Teacher, colleague, friend.
You taught me more than you know.

Cambridge University Press
978-0-521-41152-3 - Laminar Flow Analysis
David F. Rogers
Frontmatter
More information

CONTENTS

Cambridge University Press
978-0-521-41152-3 - Laminar Flow Analysis
David F. Rogers
Frontmatter
More information

PREFACE

For many years the author has been engaged in teaching fluid dynamics. At the undergraduate level, one of the most neglected subjects in fluid dynamics is viscous flow. At this level, viscous flow is frequently presented as a qualitative discussion of the characteristics of laminar and turbulent boundary layers and of the effects of separation and reverse flow. Analytical results are generally confined to the simpler exact solutions of the Navier-Stokes equations, e.g., flow in a channel. Blasius's solution of the flat plate laminar boundary layer and perhaps the von Karman momentum integral technique are briefly discussed and the results stated. Yet undergraduates are quite capable of understanding and appreciating both the underlying mathematics and the physics of classical viscous flows, *provided* they can actually *obtain* the solutions themselves.

Today, almost every undergraduate engineering student either owns or has ready access to a personal computer capable of numerically solving the governing equations for classical laminar boundary layers. This capability leads to both a new approach to teaching viscous flow and a great danger that the fundamental mathematics, modeling and analytical skills developed from a study of viscous flow will be neglected in favor of developing numerical computational skills. Just presenting a numerical technique or providing a 'canned' program to solve a problem is not enough. The student must understand the underlying physical, mathematical and modeling concepts inherent in any solution, be it analytic or numerical.

In this book the major emphasis is to present a technique of analysis which aids the formulation, the understanding and the solution of the defined class of problems. The student must first choose a mathematical model and derive the governing equations based on realistic assumptions, or he must become thoroughly aware of the limitations and assumptions associated with existing models. An appropriate solution technique is then selected. The solution technique can be either analytical or numerical. Analysis does not end once a solution is obtained; in a sense it only begins. The investigator must now interpret the results in light of the previous assumptions and the physical world. In light of these results, he may decide to change the analysis technique, remove some limiting assumptions, or even change the mathematical model or the specified boundary conditions.

Selecting the material coverage for the present volume proved difficult. Since the study of turbulent flows continues to undergo intensive investigation, the material is restricted to the analysis of laminar flows. The study of even laminar viscous flow encompasses such a wide area that it is impossible to cover it in any depth in a single volume. The laminar flows which illustrate the fundamental mathematics and physics with the least complexity are the similar flows. In discussing nonsimilar flows it was decided to use the locally nonsimilar method, since it represents a logical extension of the similar analyses, rather than introduce the approximate momentum integral techniques, singular perturbation methods, direct finite difference integration or other computational fluid dynamics techniques. These techniques are adequately covered in other texts. Both the breadth and depth of the subject matter is increased by supplementing the classical analyses with computer-aided analysis algorithms. These algorithms contribute to the understanding of many of the problems of laminar flow that are not amenable to analytical solutions. In this way problems which previously have only been formulated in the classroom can now be completely solved and analyzed.

The computer algorithms allow the reader to follow each step in the numerical solution. This is an important part of the analysis. A thorough understanding of a computer algorithm leads to a better understanding of the mathematical model and its limitations, and hence to a better physical understanding. The student can make improvements and modifications to the algorithms. Each algorithm is fully documented. Each algorithm was implemented in a higher level language and the pseudocode algorithm derived from that implementation.† The fully annotated computer runs increase student understanding. They can also be used by students to check their own implementations.

The book begins by deriving the Navier-Stokes equation for a viscous compressible variable property fluid. The second chapter considers exact solutions of the incompressible Navier-Stokes equations. In Chapter 3, the boundary layer equations are derived and the incompressible hydrodynamic boundary layer equations solved with and without mass transfer at the wall. Next, in Chapter 4 forced convection is considered. In Chapter 5 free convection is discussed, and in Chapter 6 the compressible laminar boundary layer is analyzed. Appendix A develops the fourth-order Runge-Kutta numerical integration scheme. Appendix B discusses the Nachtsheim-Swigert iteration scheme for asymptotic boundary conditions. Appendix C is an attempt to present in a unified manner the important numerical results for similar laminar boundary layers. Appendix D presents, in pseudocode suitable for implementation in any programming language, a program for solving the general similar laminar boundary layer equations. Finally, Appendix E presents a number of problems.

---

†The author recommends True Basic, an implementation of ANSI Basic, as an implementation tool. Experience has shown that ANSI Basic provides an excellent working environment for students with limited computer experience. True Basic and the True Basic implementations of the pseudocode are available from True Basic Inc., 1-800-TR-BASIC (see also Appendix D.)

A unifying mechanism among the various topics is the logical progression from simple to complex governing differential equations and boundary conditions. The similar hydrodynamic boundary layer is governed by a single nonlinear ordinary differential equation and a single asymptotic boundary condition. Forced convection is governed by a pair of uncoupled ordinary differential equations which can be solved successively. Each equation is subject to a single asymptotic boundary condition. Free convection and the similar compressible boundary layer are both governed by a pair of coupled ordinary differential equations which must be solved simultaneously. They are subject to a pair of asymptotic boundary conditions (see Section 1–10). Analogous situations occur for the locally nonsimilar method.

The problems in Appendix E are generally of three types: numerical, parametric and directed analysis. The numerical problems are generally of the form — 'Given the following specific information, calculate a required result'. This is the type of problem that is generally found in textbooks. Examples are Problems 1–4 and 4–6. The parametric problems are of the form — 'investigate the effect of the variation of a parameter on the numerical solution'. These problems are generally amenable to solutions using modifications of programs derived from the pseudocode algorithms discussed in the text and given in Appendix D. They allow the student to ask the question 'What if —— ?' and lead to a better physical understanding of the mathematical model. The author has found this type of problem very useful in teaching the concept of parametric analysis. Examples are Problems 3–4 and 4–8. The directed analysis problems seek to guide the student through an original analysis of a given problem. They are frequently skeletons of the analyses in classical papers existing in the literature. They serve to extend and diversify the material covered in any specific course. For example, there are several directed analysis problems on rotational flows which serve as an introduction to that topic. Frequently the result of a directed analysis problem is a differential equation or system of differential equations that require numerical solution. In this case, the student is shown how to put these equations in a form similar to that of the equations solved by one of the analysis algorithms. In this way, the directed analysis problems serve as a unifying mechanism for the various topics in fluid dynamics. These problems generally require considerable effort on the part of the student. It has therefore been the policy to assign various directed analysis problems to individual students at the rate of one per week. In this way it is possible, while still covering the basic material, to tailor the course to individual interests and needs, e.g., rotational flow. Examples of directed analysis problems are Problems 2–5, 3–10 and 4–11.

The prerequisites for understanding the material are a course in classical potential flow theory, gasdynamics or compressible, nonviscous, heat conducting flow, college level mathematics through classical differential equations, skill in a higher level computer language, e.g. ANSI Basic, Fortran, C or Pascal and the availability of a computer. The book is suitable for a senior-level elective course or a first year graduate course on viscous flows.

As with any book, a number of people have contributed to its existence, form and substance. First let me thank my partner and wife Nancy, who typeset the book using TeX from rough handwritten notes, roughly marked up typewritten copy and incomplete computer files. She made multiple changes first one way and then back to the original way, and frequently to still a third way, while I made up my mind about what I wanted. More than that no author nor husband can expect. Professor John Anderson deserves a note of thanks for encouraging me to finally put the book into publishable form. Special thanks are due my colleague, office mate and co-author, Professor J. Alan Adams, for first introducing me to forced and free convection flows. Finally, my very special thanks to Professor Gabriel (Gabby) Karpouzian, who read the entire manuscript, checked the equations and the cross-references and made many valuable contributions. Still, as in any book, I am sure that typos, incorrect cross-references, numerical errors, etc. remain. Those are my responsibility. When you find them please bring them to my attention.

The pedagogical approach taken in this text is such that it requires innovation and involvement on the part of both the student and professor. I hope that it is as much 'fun' for you as it is and was for me.

David F. Rogers
Annapolis, Maryland
USA