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978-0-521-41063-2 - The Selected Works of J. Frank Adams, Volume I

Edited by J. P. May and C. B. Thomas

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J. Frank Adams was one of the world's leading topologists. He solved a number of celebrated problems in algebraic topology, a subject in which he initiated many of the most active areas of research. He wrote a large number of papers during the period 1955–1988, and they are characterised by elegant writing and depth of thought. Few of them have been superseded by later work.

This selection brings together all his major research contributions. They are organised by subject matter rather than in strict chronological order. This first volume contains papers on: the cobar construction, the Adams spectral sequence, higher order cohomology operations, and the Hopf invariant one problem; applications of  $K$ -theory; generalised homology and cohomology theories. The second volume is mainly concerned with Adams' contributions to: characteristic classes and calculations in  $K$ -theory; modules over the Steenrod algebra and their Ext groups; finite  $H$ -spaces and compact Lie groups; maps between classifying spaces of compact groups.

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OF J. FRANK ADAMS  
VOLUME I

*Edited by*

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## Contents

<i>Introduction</i>	page xi
<i>Biographical data</i>	xiii
<i>Acknowledgements</i>	xv
<b>The cobar construction, the Adams spectral sequence, higher order cohomology operations, and the Hopf invariant one problem</b>	
On the chain algebra of a loop space	1
On the cobar construction	27
On the structure and applications of the Steenrod algebra	34
On the non-existence of elements of Hopf invariant one	69
<b>Applications of <math>K</math>-theory</b>	
Applications of the Grothendieck–Atiyah–Hirzebruch functor $K(X)$	154
Vector fields on spheres	161
On complex Stiefel manifolds	191
On matrices whose real linear combinations are nonsingular and correction	214
On the groups $J(X)$ —I	222
On the groups $J(X)$ —II	237
On the groups $J(X)$ —III	272
On the groups $J(X)$ —IV and correction	302
$K$ -theory and the Hopf invariant	354
Geometric dimension of bundles over $RP^n$	362
<b>Generalised homology and cohomology theories, and a survey</b>	
Lectures on generalised cohomology	377
Algebraic topology in the last decade	515

## Contents of Volume II

<i>Introduction</i>	page ix
<i>Biographical data</i>	xi
<i>Acknowledgments</i>	xiii
<b>Characteristic classes and calculations in <math>K</math>-theory</b>	
On formulae of Thom and Wu	1
On Chern characters and the structure of the unitary group	13
Chern characters revisited and addendum	24
The Hurewicz homomorphism for $MU$ and $BP$	29
Hopf algebras of cooperations for real and complex $K$ -theory	36
Operations of the $n$ th kind in $K$ -theory	60
Operations on $K$ -theory of torsion-free spaces	64
Stable operations on complex $K$ -theory	73
Primitive elements in the $K$ -theory of $BSU$	77
<b>Modules over the Steenrod algebra and their Ext groups</b>	
A finiteness theorem in homological algebra	87
A periodicity theorem in homological algebra	93
Modules over the Steenrod algebra	106
Sub-Hopf-algebras of the Steenrod algebra	118
What we don't know about $RP^\infty$	126
Calculation of Lin's Ext groups	132
The Segal conjecture for elementary abelian $p$ -groups	143
<b>Finite <math>H</math>-spaces and compact Lie groups</b>	
The sphere, considered as an $H$ -space mod $p$	169
$H$ -spaces with few cells	178
Finite $H$ -spaces and algebras over the Steenrod algebra and correction	184
Finite $H$ -spaces and Lie groups	235
$\text{Spin}(8)$ , triality, $F_4$ and all that	243
The fundamental representations of $E_8$	254
2-Tori in $E_8$	264

Cambridge University Press

978-0-521-41063-2 - The Selected Works of J. Frank Adams, Volume I

Edited by J. P. May and C. B. Thomas

Frontmatter

[More information](#)*Contents*

ix

<b>Maps between classifying spaces of compact Lie groups</b>	
Maps between classifying spaces	275
Maps between classifying spaces, II	316
Maps between classifying spaces, III	381
Maps between $p$ -completed classifying spaces	399
<b>Miscellaneous papers in homotopy theory and cohomology theory</b>	
An example in homotopy theory	404
An example in homotopy theory	406
A variant of E. H. Brown's representability theorem	408
Idempotent functors in homotopy theory	422
The Kahn–Priddy theorem	429
Uniqueness of $BSO$	440
Graeme Segal's Burnside ring conjecture	475
A generalization of the Segal conjecture	485
A generalization of the Atiyah–Segal completion theorem	500
Atomic spaces and spectra	506
<b>Two unpublished expository papers</b>	
Two theorems of J. Lannes	515
The work of M. J. Hopkins	525



Cambridge University Press

978-0-521-41063-2 - The Selected Works of J. Frank Adams, Volume I

Edited by J. P. May and C. B. Thomas

Frontmatter

[More information](#)

## Introduction

Frank Adams was a great mathematician with a fine expository style. These two volumes contain the bulk of his numerous papers, grouped according to subject, and roughly chronologically within groups. We chose this organisation because of Adams' practice of returning periodically to certain subjects dear to his heart, re-examining them in the light of intervening research. We have added no editorial material since we prefer to let Adams' own introductions to his papers speak for themselves. We have also made no attempt to note errors since Adams was a scrupulously careful author. Several of his papers have published corrections; these we have included.

Adams' final bibliography contains 82 published items. These volumes contain 52 of them. The omitted items fall into five categories. There are five early papers (to 1957), an appendix to a paper of another author, and a published letter which we feel are probably not of lasting mathematical interest. There are four announcements of results which were published elsewhere. There are three biographical items. There are fifteen primarily expository articles, most of which were published in conference proceedings; we have chosen to include four of these, which we feel are of more lasting interest than the others.

The five remaining omitted items are books. The first of these, from 1964, is *Stable Homotopy Theory, Volume 3* in the Springer Lecture Notes in Mathematics. It contains material that has been superseded mathematically by results published elsewhere. Still, it has many delightful passages, and comparison of it with later work shows how rapidly algebraic topology in general, and Adams' work in particular, was progressing in those days. A second, *Algebraic Topology: a student's guide* consists primarily of selected reprints. The remaining three are highly recommended reading. Two of them, *Lectures on Lie Groups* and *Stable Homotopy and Generalized Homology* are being kept in print by the University of Chicago Press and remain among the best references on their subjects. The third, *Infinite Loop Spaces* is Study 90 of the Annals of Mathematics, published by Princeton University Press. It is vintage Adams, beautifully and humorously written, and it contains capsule summaries of various topics in algebraic topology other than the one described by the title.

The second volume ends with two expository articles which perhaps were not intended for publication. They well illustrate Adams' interest in the work of young algebraic topologists and his success in illuminating the most recent developments in his subject.

## Biographical data

J. Frank Adams was born on 5 November 1930, in Woolwich, London, and died on 7 January 1989. He was married in 1953 and had one son and three daughters.

The main dates in his education were Bedford School until 1948, followed by military service in the Royal Engineers (1948–9) and attendance at Trinity College, Cambridge (1949–55), obtaining first class honours in Part II of the mathematical tripos (1951) and a distinction in Part III a year later. He was awarded his B.A. in 1952, his Ph.D. in 1955, his M.A. in 1956 and the higher degree of Sc.D. in 1982.

Following the completion of his Ph.D., he was appointed a Junior Lecturer at Oxford (1955–6), and he held a Research Fellowship at Trinity College, Cambridge, until 1958. During this time he also made his first visit to the University of Chicago (Summer 1957) and held a Commonwealth Fund Fellowship at the Institute of Advanced Study in Princeton (1957–8).

On returning to England he became an Assistant Lecturer at Cambridge, combining this with the Directorship of Studies in Mathematics at Trinity Hall. He was again a visiting member of the Institute at Princeton in the fall of 1961.

In 1962 Adams moved to the University of Manchester first as a Reader and then as Fielden Professor of Pure Mathematics (1964–71). In 1970 he returned to Cambridge as Lowndean Professor of Astronomy and Geometry, and he was also active as a Fellow of Trinity College.

His professional affiliations included the Association of University Teachers, the American Mathematical Society (since 1957) and the London Mathematical Society (since 1958). He was also elected a Fellow of the Royal Society in 1964 and a Foreign Associate of the National Academy of Sciences of the United States in 1985.

His frequent trips to the United States included ten lengthy visits to the University of Chicago, the last in 1985. Adams was in great demand as a speaker – his more important lectures included addresses to the International Congress of Mathematicians in Stockholm (1962) and in Moscow (1966), the Herman Weyl Lectures at the Institute in Princeton (1975) and an address to the American Mathematical Society as Bicentennial Exchange Lecturer (1976).

Over many years he contributed to the *Mathematical Reviews*, and in addition acted as referee for numerous other journals. He served as Editor, Member of the Editorial Board, or Editorial Advisor for three journals of the London Mathematical Society, the *Annals of Mathematics*, *Inventiones Mathematicae*, the *Journal of Pure and Applied Algebra*, and the *Mathematical Proceedings of the Cambridge Philosophical Society*. He also served on subcommittees to choose

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Frontmatter

[More information](#)

---

xiv

*Biographical data*

speakers in algebraic topology for the International Congress (once as Chairman), on the committee to choose Fields Medallists, and on the Consultative (programme) committee of the ICM. He gave similar organisational help to other conferences.

As a Fellow of the Royal Society he served on the mathematical sectional committee (two terms, one as chairman) and on the Council. He was also a member of the Council of the London Mathematical Society, and of the mathematics committee of the Science and Engineering Research Council (two terms).

Adams' honours include the Junior Berwick Prize (1963), the Senior Whitehead Prize (1974) and the Sylvester Medal (1982). Besides the Sc.D. from Cambridge he was also made a Doctor (h.c.) by the Universität Heidelberg in 1986.

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Frontmatter

[More information](#)

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Cambridge University Press

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Frontmatter

[More information](#)

xvi

*Acknowledgements*

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