Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

© Cambridge University Press 1991
First published 1991
Reprinted 1992
First paperback edition 1995

Library of Congress Cataloging-in-Publication data is available
A catalogue record for this book is available from the British Library

ISBN 0-521-41030-4 hardback
ISBN 0-521-49971-2 paperback

Transferred to digital printing 2003

for Jon, who made it happen
CONTENTS

Acknowledgements vii
Preface ix
Preface to the Paperback Edition xiii
1 Information 1
2 Information, Situations, and Infons 14
3 Situation Theory 49
4 Meaning and Constraints 86
5 Some Logical Issues 111
6 Mental States 145
7 Perception and Action 187
8 Situation Semantics 216
9 Topics in Situation Semantics 252
10 Retrospect and Prospect 295
References 302
Index 304
Index of symbols 308
It is therefore quite possible that we are not too far from the limits which can be achieved in artificial automata without really fundamental insights into a theory of information, although one should be very careful with such statements because they can sound awfully silly in five years.

— John von Neumann, 1949
ACKNOWLEDGEMENTS

Amongst those whose conversations and suggestions proved invaluable in the preparation of this book are Jon Barwise, Mark Crimmins, John Etchemendy, Pat Hayes, David Israel, John Perry, Stanley Peters, and other members of the STASS Research Group at CSLI, Stanford University.

David Tranah, my editor at Cambridge University Press, was always very supportive, and it was he who persuaded me that I should abandon the strictly 'textbook' style I had originally adopted, in order to make the book accessible to the much wider audience he felt would be interested in the issues I raise.

Particular thanks go to Jon Barwise. Though I began to work on the material presented here in 1985, the initial development was severely hampered by the traumas resulting from the rapid contraction forced upon the British university system by the government. Trying to feel one's way forward into uncharted territories at a time when all pressures were towards the pursuit of 'useful research' (measured in terms of how many bucks will it earn by the weekend) was not easy. There was considerable pressure to abandon work regarded as 'unproductive' (read '$'). 'Pure' research was looked upon with disdain as a luxury bought at others' expense. In my own case, my then university, Lancaster, blocked any further career advancement for me there and advised (and subsequently pressured) me to seek my future elsewhere. All in all, it was not a happy time and, without Barwise's invitation for me to spend the period 1987-9 at CSLI, it is unlikely that this book would ever have been written.

The final version of the book was completed after I had taken up the position of Carter Professor of Mathematics at Colby College in Maine, and I am particularly indebted to President Bill Cotter and Dean of Faculty Bob McArthur, who facilitated the continuation of my research after I left Stanford.
Acknowledgements

Most of all, I owe an immense debt to my wife, Jan, and daughters, Naomi and Melissa, who twice suffered the trauma of being uprooted from their home and friends, first in England and then in California, as I sought to find a means by which I could pursue my work.

Should it ever come about (and I think it will) that some of the ideas developed in these pages turn out to be of real ‘use’, I would hope that this book serves as a testament to the stupidity, even in those very terms of ‘usefulness’ that were foisted on the British university system, of judging any intellectual pursuit in terms of its immediate cash value.

Keith Devlin
Waterville, Maine
January 1991
PREFACE

Towards a mathematics of information

In Mathematics, as anywhere today, it is becoming more difficult to tell the truth ……. Telling the truth is not quite the same thing as reciting a rosary of facts. José Ortega y Gasset, in an admirable lesson summarised by Antonio Machado’s three-line poem, prophetically warned us that

the reason people so often lie is that they lack imagination: they don’t realize that the truth, too, is a matter of invention.

Sometime, in a future that is knocking at our door, we shall have to retrain ourselves or our children to properly tell the truth. The exercise will be particularly painful in mathematics. The enraptured discoveries of our field systematically conceal, like footprints erased in the sand, the analogical train of thought that is the authentic life of mathematics. Shocking as it may be to a conservative logician, the day will come when currently vague concepts such as motivation and purpose will be made formal and accepted as constituents of a revamped logic, where they will at last be allotted the equal status they deserve, side-by-side with axioms and theorems.

The above two paragraphs were not written by me, but by Gian-Carlo Rota. The words were penned on 7 February 1985, as part of the preface to the book Discrete Thoughts [11], a collection of articles on mathematics and computing assembled by Rota together with Jacob Schwartz and Mark Kac.

Rota's words provide a particularly apt opening to this book. And, it is to be hoped, add weight to what I believe to be the importance to the future of mathematics, of enterprises such as the one presented in this volume. Without a continued supply of new areas of application, mathematics would start to grow inwards, and eventually die. But fleshing

1 Professor of Applied Mathematics and Philosophy at the Massachusetts Institute of Technology, Fellow of the Los Alamos National Laboratory, the editor of the journal Advances in Mathematics, and a member of the United States National Academy of Sciences.
Preface

out and developing new mathematical tools is no easy matter, and the fumbling beginnings are all too easily dismissed as futile. The eloquent words of Professor Rota can remind us all, both the outside sceptics and the committed researchers (who also, let it be said, experience periodic doubts about the outcome of their work), that it is only by dreaming, and then striving to turn those dreams into reality, that mankind progresses.

None of which is to say that we can get it right first time. The theory outlined in these pages marks an attempt to develop some new mathematics, a mathematics of information, but only time will tell whether or not this is the ‘right’ way to do the job. It is always difficult to strive out into something new. Rota has something to say about this as well. In Chapter 1 of the same volume, he begins:

Of some fields it is difficult to tell whether they are sound or phony. Perhaps they are both. Perhaps the decision depends on the circumstances, and it changes with time. At any rate, it is not an objective fact like ‘the moon is made of green cheese’. Some subjects start out with impeccable credentials, catastrophe theory, for instance, and then turn out to resemble a three-dollar bill. Others, like dynamic programming, have to overcome a questionable background before they are reluctantly recognized to be substantial and useful. It’s a tough world, even for the judgement pronouncers.

What then, of the ideas set out (in a fumbling, embryonic form) in this essay? Where do they lie in Rota’s spectrum: sound or phony? Obviously, it is my belief that the ideas are (or rather will evolve to be) both sound and (profoundly) useful. But I cannot claim to know this for a fact. The territory is uncharted, with only a handful of travellers making those first few tentative steps. But then, is not that the very nature of true research?

One further quote, this time from the great English mathematician J. E. Littlewood [6, p.144]:

Most of the best work starts out in a hopeless muddle and floundering, sustained on the ‘smell’ that something is there. … In a new subject (or new to oneself) there can be a long preliminary process of getting to the essential core. At this process a first-rate mathematician is little, if at all, better than a PhD student. … With a collection of really difficult problems, nothing happens in a year; much happens in ten years.

After reading this essay, the reader may judge for herself how far we

---

2 At the present time, there is always the vexing issue of how to represent the third-person singular in a gender-neutral way. In this book I take the approach of using both ‘he’ and ‘she’ interchangeably in a more or less random manner.
Preface

have progressed towards the goal outlined in Chapter 1. And how much further we have still to go.

What this book is not

This book is perhaps a little out of the ordinary — certainly for a book written by a mathematician, intended to be a ‘mathematics book’. (Where are the pages of symbolic expressions?) If you have picked up the book and got this far, you will, I hope, be sufficiently intrigued to proceed further. What are you likely to find? Just what nature of book do you have in your hands? The best way to answer that is to sit down and read it. But here, for the impatient, a brief guide to what the book is not, and what it is.

This book is not intended to be a work of scholarship. That is, I did not set out to examine other work in this general area, or to compare the theory developed here with any other theories. Others are free to do that if they wish. That was not my intention.

This book is not a ‘philosophy text’. True enough, a lot of the topics dealt with are regular fare on the philosopher’s table, and I pay considerable attention to many philosophical issues. But as an attempt to develop a tolerably useful piece of mathematics, the treatment of many deep philosophical issues is of necessity a ‘naïve’ one.

This book is not a ‘linguistics text’. Despite the large amount of space dedicated to natural language semantics, my interest in natural language is its great power, versatility, and robustness as a vehicle for conveying information. As with the philosophy, so too is the linguistics you will find here essentially ‘naïve’.

This book is not a ‘computer science text’. Though the issues dealt with are all central to computer science (especially the information processing side of the subject), and though I utilize a number of concepts from computer science and occasionally use computer science examples to illustrate various points, no attention is paid to questions of implementation or computational tractability.

This book does not pretend to present a completed body of work. It is very much an account of work in progress, work that has a long way to go until its (hoped for) completion.

What this book is

So now you know what this book is not. What then, is it?
Preface

It is a mathematics book, or at least a 'pre-mathematics' book that covers issues of crucial importance to philosophers, linguists, computer scientists, and cognitive scientists. Accordingly, it has been written in a fashion that, I hope, makes it accessible to workers in all of these areas, and possibly other fields as well.

It is applied mathematics. More precisely the book describes an instance of the important (applied) mathematical activity of modeling.

It is a 'research monograph' in the spirit of mathematical research. That is to say, the goal is the development of a mathematical theory, a piece of mathematics. And very much in the spirit of present-day mathematical research, little (in fact no) attempt has been made to turn it into a work of scholarship.

The overall goal is to provide the mathematics required for a science of information. I start the process of fleshing out a mathematical framework that will (I hope) form the backbone of such a science in the same way that parts of present-day mathematics support, say, physics.

It is a challenge. By sending this volume out into the world at this early stage in the development, I hope that others will be able to progress still further.
PREFACE TO THE PAPERBACK EDITION

The first edition of Logic and Information was completed in the fall of 1990. Since my book was, in many ways, a sequel to Barwise and Perry's Situations and Attitudes, which was published in 1983, a comparison of the two volumes indicates the dramatic advances made in situation theory in the seven years that separated their appearances. In Situations and Attitudes, there were a lot of great ideas about language and cognition, but the barest hint of a mathematical theory of information to come. In 1983, a lot of significant choices remained to be made before anything like the desired theory of information would become a reality. By fall of 1990, many of those choices had been made, and situation theory was beginning to look, if not mathematical, then certainly 'pre-mathematical'. And it was as a 'pre-mathematics' book that Logic and Information was presented. A sequel volume was promised, in which some of the details of the mathematical theory would be described.

That sequel volume has not yet been written, at least by me, though enough of the mathematical details have been worked out and published by various other people to more than fill a book. In my own case, as a result of one of those serendipitous events that so often shape our lives, I was sidetracked. In 1991, I began an exciting research collaboration with Duska Rosenberg, a social scientist turned computer scientist, in which we found applications of situation theory in sociolinguistics and in the analysis of certain kinds of written documents as carriers of information. Though not designed for application in such a domain, situation theory turned out to be a powerful analytic tool for analyzing certain kinds of linguistic data, and the results of our collaboration have appeared in a number of research publications, among them:

Preface to the Paperback Edition


The one further piece of relevant mathematics that I did publish was the inclusion of a chapter on non-well-founded set theory in my book The Joy of Sets, published by Springer-Verlag in 1992.

For readers who want to pursue the mathematical development of situation theory, there are a number of relevant volumes in the CSLI Lecture Notes series, listed below in chronological order:


Other sources for the mathematics of situation theory are:


For the ‘real’ mathematical sequel to Logic and Information, you will have to await the publication of the book.
Preface to the Paperback Edition


For a more comprehensive treatment of non-well-founded set theory than is given in my own The Joy of Sets, there is the forthcoming book:


Continuing the collaboration begun with their book The Liar (Oxford University Press, 1987), Barwise and Etchemendy used ideas of situation theory in the design of their educational software package Hyperproof, published by CSLI Publications in 1995. More recently, the two have co-authored two further articles:


The foundational mathematical work of Aczel and Lunnon that began with their papers in Volumes 2 and 4 of Situation Theory and its Applications has led to a more recent paper:


Though it does not deal specifically with situation theory, the work of Gabbay on what he calls 'labelled deductive systems' is highly relevant. This theory, which in some ways can be thought of as a proof-theoretic analog of situation theory, is described in a number of publications, including:


The connections between Gabbay's work and situation theory are dealt with in:

Preface to the Paperback Edition


Another mathematical approach to information that, while superficially different, is at a deeper level fairly similar to situation theory in some respects is described in


There has, then, been quite a lot of activity in situation theory since Logic and Information first appeared. Some of that activity has been mathematical in nature, some has been in the area of applications. On the other hand, the basic ideas of situation theory have not changed during this period, as they did in the years immediately following the publication of Situations and Attitudes, which is one of the reasons why the publication of this paperback edition seemed a useful thing to do.

Will there in fact be a Logic and Information II? I have certainly not ruled out the possibility. Right now, I am fully occupied following the unanticipated path of applications of situation theory in sociolinguistics and systems design. And besides, the forthcoming Barwise–Seligman book does much of what was promised by way of a sequel volume. But given the steady progress in situation theory and its applications, I may yet be tempted to fulfill the promise I made in 1991.

In preparing this edition, the only changes I made to the text were the corrections of some minor typos in the first edition.

Keith Devlin
Moraga, California
April 1995