This is an advanced text for higher degree materials science students and researchers concerned with the strength of highly brittle covalent–ionic solids, principally ceramics. It is a reconstructed and greatly expanded edition of a book first published in 1975.

The book presents a unified continuum, microstructural and atomistic treatment of modern-day fracture mechanics from a materials perspective. Particular attention is directed to the basic elements of bonding and microstructure that govern the intrinsic toughness of ceramics. These elements hold the key to the future of ceramics as high-technology materials – to make brittle solids strong, we must first understand what makes them weak. The underlying theme of the book is the fundamental Griffith energy-balance concept of crack propagation. The early chapters develop fracture mechanics from the traditional continuum perspective, with attention to linear and nonlinear crack-tip fields, equilibrium and non-equilibrium crack states. It then describes the atomic structure of sharp cracks, the topical subject of crack–microstructure interactions in ceramics, with special focus on the concepts of crack-tip shielding and crack-resistance curves, and finally deals with indentation fracture, flaws, and structural reliability.

Brittle fracture crosses the boundaries between materials science, structural engineering, and physics and chemistry. This book develops a cohesive account by emphasising basic principles rather than detailed factual information. Due regard is given to model brittle materials such as silicate glass and polycrystalline alumina, as essential groundwork for ultimate extension of the subject matter to more complex engineering materials.

This book will be used by advanced undergraduates, beginning graduate students and research workers in materials science, mechanical engineering, physics and earth science departments interested in the brittle fracture of ceramic materials.

Fracture of brittle solids

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BRIAN LAWN

NIST Fellow

Fracture of brittle solids

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Preface

This book is a restructured version of a first edition published in 1975. As before, the objective is a text for higher degree students in materials science and researchers concerned with the strength and toughness of brittle solids. More specifically, the aim is to present fracture mechanics in the context of the 'materials revolution', particularly in ceramics, that is now upon us. Thus whereas some chapters from the original are barely changed, most are drastically rewritten, and still others are entirely new.

Our focus, therefore, is 'brittle ceramics'. By brittle, we mean cracks of atomic sharpness that propagate essentially by bond rupture. By ceramics, we mean covalent–ionic materials of various persuasions, including glasses, polycrystalline aggregates, minerals, and even composites. Since 1975, our knowledge of structural ceramics has equalled, some would insist surpassed, that of metals and polymers. But it is brittleness that remains the singular limiting factor in the design of ceramic components. If one is to overcome this limitation, it is necessary first to understand the underlying mechanics and micromechanics of crack initiation and propagation. Prominent among improvements in this understanding have been a continuing evolution in the theories of continuum fracture mechanics and new conceptions of fundamental crack-tip laws. Most significant, however, is the advent of 'microstructural shielding' processes, as manifested in the so-called crack-resistance- or toughness-curve, with far-reaching consequences in relation to strength and toughness. This developing area promises to revolutionise traditional attitudes toward properties design and processing strategies for ceramics.

The unifying theme of the book is the thermodynamic energy-balance concept expounded by Griffith in his classic 1920 paper. Griffith's concept leads naturally to classifications of crack systems as equilibrium or dynamic, stable or unstable, reversible or irreversible. His concept survives because of its inherent generality: in proceeding to more complex systems one needs only to modify existing terms, or add new ones, in the expression for the total energy of the crack system. All soundly-based fracture theories derive either directly from the Griffith concept or from some alternative concept with underlying equivalence, such as Irwin's stress-intensity factor.

In attempting to construct an integrated picture of fracture, one becomes aware of widely diverse perspectives on brittle cracks. Most traditional is the 'global' perspective of the engineer, who sees cracks in terms of a slit continuum, treating the tip and its surrounds as a singular (black box) zone. At the opposite end of the spectrum is the crack-tip 'enclave' perspective of the physicist–chemist, who defines the processes of discrete bond rupture in terms of intersurface force functions. Both viewpoints are valuable: the first gives us general parameters such as mechanical-energy-release rate G and stress-intensity factor K for quantifying the 'motive' for fracture in terms of extraneous variables like applied loads, specimen geometry, environmental concentration, etc.; the second provides us with a basis for describing the fundamental structure of atomically sharp cracks and thereby defining laws of extension. And now we must add a relatively new perspective, that of the materials scientist, who seeks to incorporate discrete dissipative elements into ceramic microstructures in order to overcome the intrinsic brittleness. It is at this level that the concept of shielding emerges, in the form of an intervening dissipative zone which screens the crack-tip enclave from the external applied loads. Innovations in microstructural shielding processes hold the key to the next generation of strong and tough brittle materials.

As with any attempt to tie these disparate perspectives into a cohesive description, it is inevitable that conflicts in notation will arise. In seeking compromise I have leant toward materials terminology. Among the more conspicuous symbols is the Griffith c rather than the solid mechanics a for crack size. Also notable are the symbols for toughness, R and T, in place of the engineering parameters G_R and K_R; the former serve to emphasise that the intrinsic resistance to crack propagation is an equilibrium material property, ultimately expressible as an integral of a constitutive stress–displacement relation without reference to fracture at all.

The layout of the book follows a loose progression from scientific fundamentals at one end to engineering design at the other. Historical and conceptual foundations are laid in chapter 1, with a review of the energy-balance concept and flaw hypothesis of Griffith. Chapters 2 and 3 develop a theoretical description of crack propagation in terms of continuum fracture mechanics, with an emphasis on equilibrium configurations.

Chapters 4 and 5 extend these considerations to moving cracks, dynamic ('fast') and kinetic ('slow'), with special attention in the latter case to environmental chemistry. In chapter 6 we analyse crack-tip processes at the atomic level, again with provision to include chemistry in the fundamental crack laws. Chapter 7 considers the influence of microstructure on the fracture mechanics, with accent on some of the promising shielding mechanisms that are emerging in the toughness description. One of the most powerful and widespread methodologies for evaluating ceramic materials, indentation fracture, is surveyed in chapter 8. In chapter 9 we deal with the issue of flaws and crack initiation. Finally, in chapter 10, strength and reliability are addressed.

An understanding of fracture mechanics is best obtained by concentrating on basic principles rather than on factual information. Consequently, our attention to 'model' materials like homogeneous glass and polycrystalline alumina should be seen as essential groundwork for ultimate extension to more complex engineering materials. That philosophy extends to the literature citations. We have not sought to provide an extensive reference list, but rather a selective bibliography. It is a hope that, in an age where the published word is fast becoming a lost forum of communication, the reader will be persuaded to consult the open literature.

Many colleagues and students have contributed greatly to this venture. Special mention is due to Rodney Wilshaw, former co-author and old friend, with whom the first edition was conceived and produced. Soon after publication of that earlier version Rod turned from academic endeavours to a life on the land. He gracefully withdrew his name from the cover of this edition. His spirit is nevertheless still to be found in the ensuing pages. Other major contributors over the years include: S. J. Bennison, L. M. Braun, S. J. Burns, H. M. Chan, P. Chantikul, R. F. Cook, T. P. Dabbs, F. C. Frank, E. R. Fuller, B. J. Hockey, R. G. Horn, S. Lathabai, Y.-W. Mai, D. B. Marshall, N. P. Padture, D. H. Roach, J. Rödel, J. E. Sinclair, M. V. Swain, R. M. Thomson, K.-T. Wan and S. M. Wiederhorn. I also thank R. W. Cahn for his encouragement to embark on this second edition, and his perseverance during its completion. Finally, to my wife Valerie, my heartfelt appreciation for enduring it all.

Brian Lawn

Glossary of symbols and abbreviations

SI units are used throughout, with the following prefixes:

k	kilo	10^3	m	milli	10^{-3}
M	mega	10^6	μ	micro	10^{-6}
G	giga	10^9	n	nano	10^{-9}
T	tera	10^{12}	p	pico	10^{-12}
			f	femto	10^{-15}
			a	atto	10^{-18}

Symbols (with units)

a	inclusion or pore radius (μm); characteristic contact radius (μm)
a_c	critical contact size (μm)
a_0	atomic spacing (nm)
A	cross-sectional area (mm²); Auerbach constant
b	minor axis in Inglis elliptical cavity (μm); magnitude of Burgers vector (nm)
b_0	lattice spacing (nm)
c	characteristic crack size (μm)
c_B	crack size at branching (μm)
c_C	critical crack size (μm)
c_f	flaw size (μm)
c_F	crack size at failure (μm)
c_I	crack size at pop-in (μm)
c_M	crack size at activated failure (μm)

c_0	starter crack (notch) size (mm)
C	crack area (μm^2)
d	beam thickness (mm); characteristic spacing between microstructural elements (μm)
E	Young's modulus (GPa)
E'	E, plane stress; $E/(1-\nu^2)$, plane strain (GPa)
F	line force (force per unit length) (N m^{-1})
F_B	force on stretched atomic bond (nN)
F_n	lattice-modified force (nN)
ΔF	activation free energy (aJ molec^{-1})
f_i	angular function in crack-tip displacement field
f_{ij}	angular function in crack-tip stress field
$\mathscr{G}$	net crack-extension force, or 'motive' (J m^{-2})
G	mechanical-energy-release rate (J m^{-2})
G_A	global mechanical-energy-release rate (J m^{-2})
G_C	critical mechanical-energy-release rate (J m^{-2})
$G_\mathbf{R}$	G_A in material with shielding (J m^{-2})
G_*	crack-tip enclave mechanical-energy-release rate (J m^{-2})
G_μ	shielding-zone mechanical-energy-release rate (J m^{-2})
G_0	cohesion-zone mechanical-energy-release rate (J m^{-2})
h	cantilever-beam crack-opening displacement (μm)
$\mathbf{h}$	Planck constant (6.6256×10^{-34} J s)
H	indentation hardness (GPa)
J	Rice line integral (J m^{-2})
k	elastic coefficient for Hertzian contact
$\mathbf{k}$	Boltzmann constant (1.3805×10^{-23} J K^{-1})
$\mathscr{k}$	net K-field at singular tip (MPa m$^{1/2}$)
K	stress-intensity factor (MPa m$^{1/2}$)
K_A	global stress-intensity factor (MPa m$^{1/2}$)
K_B	stress-intensity factor at crack branching (MPa m$^{1/2}$)
K_C	critical stress-intensity factor (MPa m$^{1/2}$)
K_R	residual stress-intensity factor (MPa m$^{1/2}$)
$K_\mathbf{R}$	K_A in material with shielding (MPa m$^{1/2}$)
K_μ	shielding-zone stress-intensity factor (MPa m$^{1/2}$)
K_*	crack-tip enclave stress-intensity factor (MPa m$^{1/2}$)

K_0	cohesion-zone stress-intensity factor (MPa $m^{1/2}$)
K_I, K_{II}, K_{III}	mode I, II, III stress-intensity factors (MPa $m^{1/2}$)
l	beam span in flexure specimen (mm); grain size (μm)
l_C	critical grain size for spontaneous microcracking (μm)
L	bridging zone length (mm); specimen dimension (mm)
m	molecular mass (10^{-27} kg)
n	crack velocity power-law exponent; number of atoms in lattice-crack chain
p_C^B	critical bridging stress (MPa)
p_C^P	critical fibre pullout stress (MPa)
p^D	fibre debonding stress (MPa)
p_E	environmental gas pressure (kPa)
p_{Th}	theoretical cohesive stress (GPa)
p_γ	cohesive surface stress at crack interface (GPa)
p_μ	microstructural shielding tractions at crack interface (MPa)
p_0	mean contact pressure (MPa)
P	applied point load, contact load (N)
P_C	critical contact load (N)
P_+, P_-	applied load extremes for lattice trapping (N)
P	probability of failure
Q	heat input (J)
r	radial crack-tip coordinate (μm); fibre or sphere radius (μm)
R	crack-resistance energy per unit area (J m^{-2})
R-curve	resistance-curve
R_E	crack-resistance energy in interactive environment (J m^{-2})
R_μ	microstructural shielding component of resistance energy (J m^{-2})
R_0	crack-resistance energy in a vacuum (J m^{-2})
R_∞	steady-state crack-resistance energy (J m^{-2})
R^+, R^-	crack-resistance trapping range (J m^{-2})
R'	quasi-equilibrium crack-resistance energy (J m^{-2})
s	arc length (m)
S	entropy (J K^{-1})

t	time (s)
t_F	time to failure (lifetime) (s)
T	toughness (MPa $m^{1/2}$)
T-curve	toughness-curve
T_E	toughness in interactive environment (MPa $m^{1/2}$)
T_μ	microstructural shielding component of toughness (MPa $m^{1/2}$)
T_0	toughness in a vacuum (MP $m^{1/2}$)
T_∞	steady-state toughness (MPa $m^{1/2}$)
$\mathbf{T}$	absolute temperature (K)
$\mathscr{T}$	traction vector in J-integral (MPa)
u	crack-opening displacement (μm); load-point displacement (μm)
$\mathbf{u}$	displacement vector (μm)
u_i	component of displacement vector (μm)
u_Z	crack-opening displacement at edge of traction zone (μm)
u_γ	crack-opening displacement in cohesion zone (nm)
U	system internal energy (J)
U_A	energy of applied loading system (J)
U_{AA}	cohesion energy of molecule A–A (fJ)
U_{AB}	energy of terminal bond A–B– (fJ)
U_B	energy of stretched cohesive bond (fJ)
U_{BB}	cohesion energy of bond –B–B– (fJ)
U_E	elastic strain energy (J)
U_i, U_f	initial, final energy states (J)
U_K	kinetic energy (J)
U_M	mechanical energy (J)
U_S	surface energy of crack area (J)
ΔU_{Ad}	adsorption energy (J m^{-2})
$\mathscr{U}$	strain energy density in J-integral (J m^{-3})
v	crack velocity (m s^{-1})
v_l	longitudinal wave velocity (km s^{-1})
v_T	terminal velocity (km s^{-1})
v_I, v_{II}, v_{III}	velocities in regions I, II, III
V_f	volume fraction

w	specimen width (mm)
w_C	critical width of frontal-zone wake (μm)
W	Dupré work of adhesion (J m^{-2})
^hW	same, for crack growth through healed interface (J m^{-2})
^vW	same, for crack growth through virgin solid (J m^{-2})
W_{AB}	work to separate unlike bodies A–B in a vacuum (J m^{-2})
W_{BB}	work to separate like bodies B–B in a vacuum (J m^{-2})
W_{BEB}	work to separate like bodies B–B in environment E (J m^{-2})
x, y, z	Cartesian coordinates for crack system (m)
X	crack-interface coordinate measured from crack tip (mm)
X_Z	crack-interface coordinate at edge of traction shielding zone (mm)
α	specimen geometry edge correction factor; activation area (nm^2 molec^{-1}); lattice spring constant (nN nm^{-1}); thermal expansion coefficient (K^{-1})
α_0	contact geometry coefficient
β	gas pressure coefficient in crack velocity equation; lattice spring constant (nN nm^{-1}); normalised radial coordinate of contact crack initiation
γ	surface or interface energy per unit area (J m^{-2})
γ_B	intrinsic ('inert') surface energy of solid body B (J m^{-2})
γ_{BE}	interfacial energy for body B in environmental medium E (mJ m^{-2})
γ_{GB}	grain boundary energy (mJ m^{-2})
γ_{hE}	fault energy for interface healed in environment (mJ m^{-2})
γ_{IB}	interphase boundary energy (mJ m^{-2})
Γ	Gibbs surface excess (nm^{-2})
Γ_B	lattice-trapping modulation factor in cohesion energy (J m^{-2})
δ	Barenblatt crack-opening displacement (nm)
ε	strain
ε^B	bridge rupture strain
ε^M	constrained microcrack-zone dilational strain
ε^T	constrained transformation-zone dilational strain

ε^{Y}	rupture strain for plastic bridge
ε_{μ}	dilational strain in frontal-zone shielding field
ζ	kink coordinate (nm)
η	order of chemical interaction
θ, ϕ	polar coordinates for crack system
θ	fractional surface adsorption coverage
κ	Knudsen attenuation factor for free molecular flow
λ	elastic compliance (m N^{-1}); Barenblatt zone length (nm)
Λ	entropy production rate (J s^{-1})
μ	friction coefficient; shear modulus (GPa)
ν	Poisson's ratio
ν_0	lattice frequency (Hz)
ξ	critical range for stress cutoff at edge of closure zone (μm)
ξ^{B}	critical cutoff range for bridge disengagement (μm)
ξ^{P}	critical cutoff range for fibre pullout (μm)
ρ	tip radius of elliptical cavity (nm); density (kg m^{-3}); radial coordinate (m)
σ	stress (MPa)
σ_A	applied uniform stress (MPa)
σ_C	stress at tip of elliptical cavity (GPa)
σ_C^{D}	critical activation stress for dislocation motion (MPa)
σ_C^{M}	critical activation stress for microcracking (MPa)
σ_C^{T}	critical activation stress for transformation (MPa)
σ_C^{Y}	yield stress (MPa)
σ_F	failure stress (MPa)
σ_{ij}	component of stress tensor (MPa)
σ_I	inert strength (MPa)
σ_M	activated failure stress (MPa)
σ_P	proof stress (MPa)
σ_R	residual stress (MPa)
σ_S	surface stress (MPa)
σ_T	tensile stress at Hertzian contact circle (MPa)
σ_{TS}	thermal shock stress (MPa)
σ_{μ}	dilational stress in frontal-zone shielding field
σ_0	Weibull scaling stress (MPa)

τ	interfacial friction stress (MPa)
Φ	indenter half-angle
χ	indentation residual-contact coefficient
ψ	crack-geometry factor

Abbreviations

CT	compact tension specimen
DCB	double-cantilever beam specimen
DT	double-torsion specimen
NDE	non-destructive evaluation
PSZ	partially stabilised zirconia
SENB	single-edge notched beam specimen
TEM	transmission electron microscope