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The Griffith concept

Most materials show a tendency to fracture when stressed beyond some critical level. This fact was appreciated well enough by nineteenth century structural engineers, and to them it must have seemed reasonable to suppose strength to be a material property. After all, it had long been established that the stress response of materials within the elastic limit could be specified completely in terms of characteristic elastic constants. Thus arose the premise of a 'critical applied stress', and this provided the basis of the first theories of fracture. The idea of a well-defined stress limit was (and remains) particularly attractive in engineering design; one simply had to ensure that the maximum stress level in a given structural component did not exceed this limit.

However, as knowledge from structural failures accumulated, the universal validity of the critical applied stress thesis became more suspect. The fracture strength of a given material was not, in general, highly reproducible, in the more brittle materials fluctuating by as much as an order of magnitude. Changes in test conditions, e.g. temperature, chemical environment, load rate, etc., resulted in further, systematic variations in strengths. Moreover, different material types appeared to fracture in radically different ways: for instance, glasses behaved elastically up to the critical point, there to fail suddenly under the action of a tensile stress component, while many metallic solids deformed extensively by plastic flow prior to rupture under shear. The existing theories were simply incapable of accounting for such disparity in fracture behaviour.

This, then, was the state of the subject in the first years of the present century. It is easy to see now, in retrospect, that the inadequacy of the critical stress criterion lay in its empirical nature: for the notion that a solid should break at a characteristic stress level, however intuitively appealing, is not based on sound physical principles. There was a need to take a closer look at events within the boundaries of a critically loaded solid. How, for example, are the applied stresses transmitted to the inner regions where fracture actually takes place? What is the nature of the fracture mechanism itself? The answers to such questions were to hold the key to an understanding of all fracture phenomena.

The breakthrough came in 1920 with a classic paper by A. A. Griffith. Griffith considered an isolated crack in a solid subjected to an applied stress, and formulated a criterion for its extension from the fundamental energy theorems of classical mechanics and thermodynamics. The principles laid down in that pioneering work, and the implications drawn from those principles, effectively foreshadowed the entire field of present-day fracture mechanics. In our introductory chapter we critically analyse the contributions of Griffith and some of his contemporaries. This serves to introduce the reader to many of the basic concepts of fracture theory, and thus to set the scene for the remainder of the book.

1.1 Stress concentrators

An important precursor to the Griffith study was the stress analysis by Inglis (1913) of an elliptical cavity in a uniformly stressed plate. His analysis showed that the local stresses about a sharp notch or corner could rise to a level several times that of the applied stress. It thus became apparent that even submicroscopic flaws might be potential sources of weakness in solids. More importantly, the Inglis equations provided the first real insight into the mechanics of fracture; the limiting case of an infinitesimally narrow ellipse might be considered to represent a *crack*.

Let us summarise briefly the essential results of the Inglis analysis. We consider in fig. 1.1 a plate containing an elliptical cavity of semi-axes b, c, subjected to a uniform applied tension σ_A along the Y-axis. The objective is to examine the modifying effect of the hole on the distribution of stress in the solid. If it is assumed that Hooke's law holds everywhere in the plate, that the boundary of the hole is stress-free, and that b and c are small in comparison with the plate dimensions, the problem reduces to a relatively straightforward exercise in linear elasticity theory. Although the mathematical treatment becomes somewhat unwieldy, involving as it does the use of elliptical coordinates, some basic results of striking simplicity emerge from the analysis.

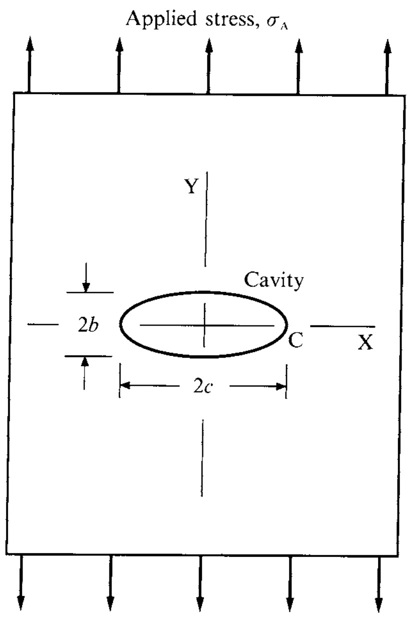

Fig. 1.1. Plate containing elliptical cavity, semi-axes b, c, subjected to uniform applied tension σ_A. C denotes 'notch tip'.

Beginning with the equation of the ellipse,

$$x^2/c^2 + y^2/b^2 = 1, \tag{1.1}$$

one may readily show the radius of curvature to have a minimum value

$$\rho = b^2/c, \quad (b < c) \tag{1.2}$$

at C. It is at C that the greatest concentration of stress occurs:

$$\begin{aligned} \sigma_C &= \sigma_A(1 + 2c/b) \\ &= \sigma_A[1 + 2(c/\rho)^{1/2}]. \end{aligned} \tag{1.3}$$

For the interesting case $b \ll c$ this equation reduces to

$$\sigma_C/\sigma_A \simeq 2c/b = 2(c/\rho)^{1/2}. \tag{1.4}$$

The ratio in (1.4) is an elastic *stress-concentration factor*. It is immediately evident that this factor can take on values much larger than unity for narrow holes. We note that the stress concentration depends on the *shape* of the hole rather than the *size*.

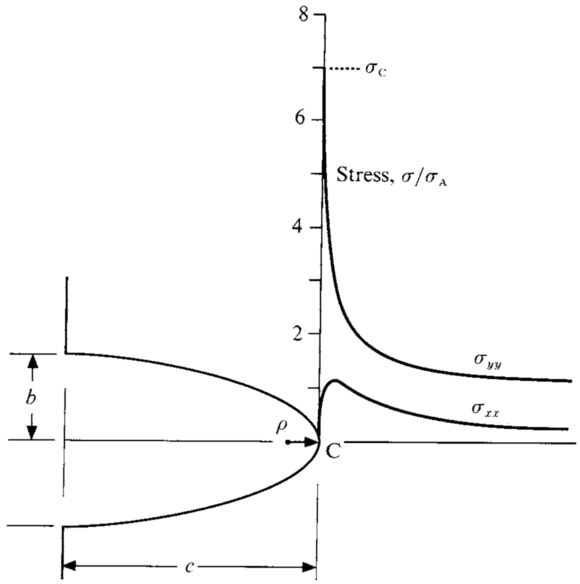

Fig. 1.2. Stress concentration at elliptical cavity, $c = 3b$. Note that concentrated stress field is localised within $\approx c$ from tip, highest gradients within $\approx \rho$.

The variation of the local stresses along the X-axis is also of interest. Fig. 1.2 illustrates the particular case $c = 3b$. The stress σ_{yy} drops from its maximum value $\sigma_C = 7\sigma_A$ at C and approaches σ_A asymptotically at large x, while σ_{xx} rises to a sharp peak within a small distance from the stress-free surface and subsequently drops toward zero with the same tendency as σ_{yy}. The example of fig. 1.2 reflects the general result that significant perturbations to the applied stress field occur only within a distance $\approx c$ from the boundary of the hole, with the greatest gradients confined to a highly localised region of dimension $\approx \rho$ surrounding the position of maximum concentration.

Inglis went on to consider a number of stress-raising configurations, and concluded that the only geometrical feature that had a marked influence on the concentrating power was the highly curved region where the stresses were actually focussed. Thus (1.4) could be used to estimate the stress-concentration factors of such systems as the surface notch and surface step in fig. 1.3, with ρ interpreted as a characteristic radius of curvature and c as a characteristic notch length. A tool was now available for appraising the potential weakening effect of a wide range of structural irregularities, including, presumably, a real crack.

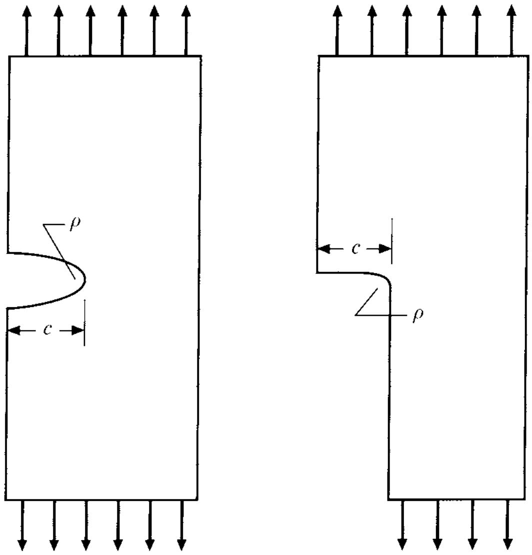

Fig. 1.3. Stress concentration half-systems: surface cavity and surface step of characteristic length c and notch radius ρ.

Despite this step forward the fundamental nature of the fracture mechanism remained obscure. If the Inglis analysis were indeed to be applicable to a crack system, then why in practice did large cracks tend to propagate more easily than small ones? Did not such behaviour violate the size-independence property of the stress-concentration factor? What is the physical significance of the radius of curvature at the tip of a real crack? These were some of the obstacles which stood between the Inglis approach and a fundamental criterion for fracture.

1.2 Griffith energy-balance concept: equilibrium fracture

Griffith's idea was to model a static crack as a reversible thermodynamic system. The important elements of the system are defined in fig. 1.4: an elastic body B containing a plane-crack surface S of length c is subjected to loads applied at the outer boundary A. Griffith simply sought the configuration that minimised the total free energy of the system; the crack would then be in a state of equilibrium, and thus on the verge of extension.

The first step in the treatment is to write down an expression for the total energy U of the system. To do this we consider the individual energy terms that are subject to change as the crack is allowed to undergo virtual

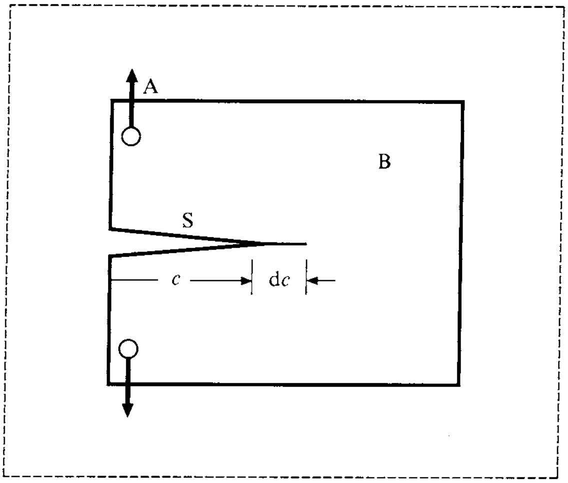

Fig. 1.4. Static plane-crack system, showing incremental extension of crack length c through dc: B, elastic body; S, crack surface; A, applied loading.

extension. Generally, the system energy associated with crack formation may be partitioned into *mechanical* or *surface* terms. The mechanical energy itself consists of two terms, $U_M = U_E + U_A$: U_E is the strain potential energy stored in the elastic medium; U_A is the potential energy of the outer applied loading system, expressible as the negative of the work associated with any displacements of the loading points. The term U_S is the free energy expended in creating the new crack surfaces. We may therefore write

$$U = U_M + U_S. \tag{1.5}$$

Thermodynamic equilibrium is then attained by balancing the mechanical and surface energy terms over a virtual crack extension dc (fig. 1.4). It is not difficult to see that the mechanical energy will generally *decrease* as the crack extends ($dU_M/dc < 0$). For if the restraining tractions across the incremental crack boundary dc were suddenly to relax, the crack walls would, in the general case, accelerate outward and ultimately come to rest in a new configuration of lower energy. On the other hand, the surface energy term will generally *increase* with crack extension, since cohesive forces of molecular attraction across dc must be overcome during the creation of the new fracture surfaces ($dU_S/dc > 0$). Thus the first term in (1.5) favours crack extension, while the second opposes it. This is the

Griffith energy-balance concept, a formal statement of which is given by the *equilibrium* requirement

$$\mathrm{d}U/\mathrm{d}c = 0. \tag{1.6}$$

Here then was a criterion for predicting the fracture behaviour of a body, firmly rooted in the laws of energy conservation. A crack would extend or retract reversibly for small displacements from the equilibrium length, according to whether the left-hand side of (1.6) were negative or positive. This criterion remains the building block for all brittle fracture theory.

1.3 Crack in uniform tension

The Griffith concept provided a fundamental starting point for any fracture problem in which the operative forces could be considered to be conservative. Griffith sought to confirm his theory by applying it to a real crack configuration. First he needed an elastic model for a crack, in order to calculate the energy terms in (1.5). For this he took advantage of the Inglis analysis, considering the case of an infinitely narrow elliptical cavity ($b \to 0$, fig. 1.1) of length $2c$ in a remote, uniform tensile stress field σ_A. Then, for experimental verification, he had to find a well-behaved, 'model' material, isotropic and closely obeying Hooke's law at all stresses prior to fracture. Glass was selected as the most easily accessible material satisfying these requirements.

In evaluating the mechanical energy of his model crack system Griffith invoked a result from linear elasticity theory (cf. sect. 2.2), namely that for any body under constant applied stress during crack formation,

$$U_A = -2U_E, \quad \text{(constant load)} \tag{1.7}$$

so that $U_M = -U_E$. The negative sign indicates a mechanical energy *reduction* on crack formation. Then from the Inglis solution of the stress and strain fields the strain energy density is readily computed for each volume element about the crack. Integrating over dimensions large compared with the length of the crack then gives, for unit width along the crack front,

$$U_E = \pi c^2 \sigma_A^2 / E' \tag{1.8}$$

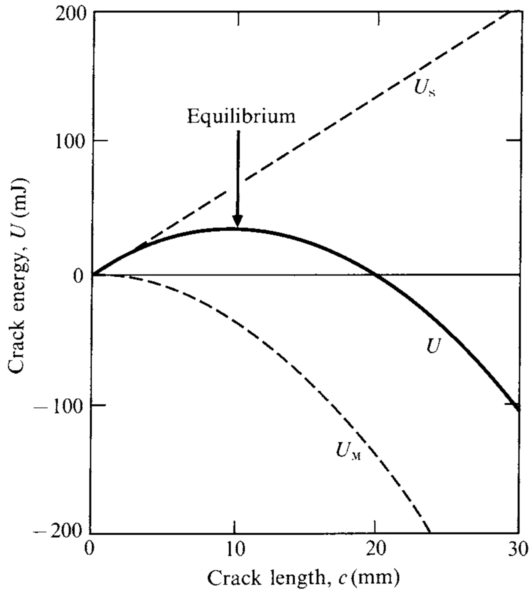

Fig. 1.5. Energetics of Griffith crack in uniform tension, plane stress. Data for glass from Griffith: $\gamma = 1.75\,\mathrm{J\,m^{-2}}$, $E = 62\,\mathrm{GPa}$, $\sigma_A = 2.63\,\mathrm{MPa}$ (chosen to give equilibrium at $c_0 = 10$ mm).

where E' identifies with Young's modulus E in plane stress ('thin' plates) and $E/(1-\nu^2)$ in plane strain ('thick' plates), with ν Poisson's ratio. The application of additional loading parallel to the crack plane has negligible effect on the strain energy terms in (1.8). For the surface energy of the crack system Griffith wrote, again for unit width of front,

$$U_S = 4c\gamma \tag{1.9}$$

with γ the free surface energy per unit area. The total system energy (1.5) becomes

$$U(c) = -\pi c^2 \sigma_A^2/E' + 4c\gamma. \tag{1.10}$$

Fig. 1.5 shows plots of the mechanical energy $U_M(c)$, surface energy $U_S(c)$, and total energy $U(c)$. Observe that, according to the Inglis treatment, an edge crack of length c (limiting case of surface notch, $b \to 0$, fig. 1.2) may be considered to possess very nearly one-half the energy of an internal crack of length $2c$.

The Griffith equilibrium condition (1.6) may now be applied to (1.10).

We thereby calculate the critical conditions at which 'failure' occurs, $\sigma_A = \sigma_F$, $c = c_0$, say:

$$\sigma_F = (2E'\gamma/\pi c_0)^{1/2}. \tag{1.11}$$

As we see from fig. 1.5, or from the negative value of d^2U/dc^2, the system energy is a maximum at equilibrium, so the configuration is *unstable*. That is, at $\sigma_A < \sigma_F$ the crack remains stationary at its original size c_0; at $\sigma_A > \sigma_F$ it propagates spontaneously without limit. Equation (1.11) is the famous Griffith strength relation.

For experimental confirmation, Griffith prepared glass fracture specimens from thin round tubes and spherical bulbs. Cracks of length 4–23 mm were introduced with a glass cutter and the specimens annealed prior to testing. The hollow tubes and bulbs were then burst by pumping in a fluid, and the critical stresses determined from the internal fluid pressure. As predicted, only the stress component normal to the crack plane was found to be important; the application of end loads to tubes containing longitudinal cracks had no detectable effect on the critical conditions. The results could be represented by the relation

$$\sigma_F c_0^{1/2} = 0.26 \text{ MPa m}^{1/2}$$

with a scatter $\approx 5\%$, thus verifying the essential *form* of $\sigma_F(c_0)$ in (1.11).

If we now take this result, along with Griffith's measured value of Young's modulus, $E = 62$ GPa, and insert into (1.11) at plane stress, we obtain $\gamma = 1.75$ J m^{-2} as an estimate of the surface energy of glass. Griffith attempted to substantiate his model by obtaining an independent estimate of γ. He measured the surface tension within the temperature range 1020–1383 K, where the glass flows easily, and extrapolated linearly back to room temperature to find $\gamma = 0.54$ J m^{-2}. Considering that even present-day techniques are barely capable of measuring surface energies of solids to very much better than a factor of two, this 'agreement' between measured values is an impressive vindication of the Griffith theory.

1.4 Obreimoff's experiment

Plane cracks in uniform tension represent just one application of the energy-balance equation (1.6). To emphasise the generality of the Griffith concept we digress briefly to discuss an important experiment carried out

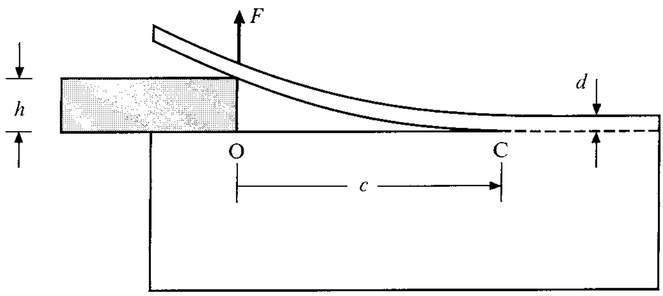

Fig. 1.6. Obreimoff's experiment on mica. Wedge of thickness h inserted to peel off cleavage flake of thickness d and width unity. In this configuration both crack origin O and tip C translate with wedge.

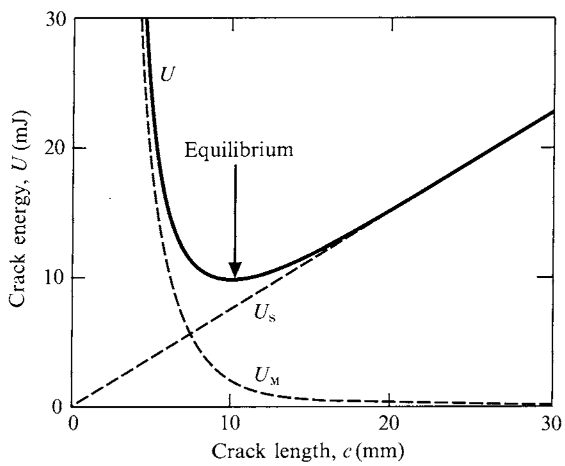

Fig. 1.7. Energetics of Obreimoff crack. Data for mica from Obreimoff: $\gamma = 0.38\ \mathrm{J\ m^{-2}}$ (air), $E = 200$ GPa, $h = 0.48$ mm, $d = 75\ \mu$m (chosen to give equilibrium at $c_0 = 10$ mm).

by Obreimoff (1930) on the cleavage of mica. This second example provides an interesting contrast to the one treated by Griffith, in that the equilibrium configuration is *stable*.

The basic arrangement used by Obreimoff is shown in fig. 1.6. A glass wedge of thickness h is inserted beneath a thin flake of mica attached to a parent block, and is made to drive a crack along the cleavage plane. In this case we may determine the energy of the crack system by treating the cleavage lamina as a freely loaded cantilever, of thickness d and width unity, built-in at the crack front distant c from the point of application of the wedge. We note that on allowing the crack to form under constant