
CONTENTS OF VOLUME 1

<i>Contents of Volume 2</i>	viii
<i>Preface</i>	xi
1 Linear transformations of the plane	1
1.1 Affine planes and vector spaces	1
1.2 Vector spaces and their affine spaces	7
1.3 Functions and affine functions	13
1.4 Euclidean and affine transformations	16
1.5 Linear transformations	18
1.6 The matrix of a linear transformation	21
1.7 Matrix multiplication	22
1.8 Matrix algebra	24
1.9 Areas and determinants	26
1.10 Inverses	32
1.11 Singular matrices	36
1.12 Two-dimensional vector spaces	39
Appendix: the fundamental theorem of affine geometry	43
Summary	45
Exercises	46
 2 Eigenvectors and eigenvalues	 55
2.1 Conformal linear transformations	55
2.2 Eigenvectors and eigenvalues	58
2.3 Markov processes	66
Summary	72
Exercises	73
 3 Linear differential equations in the plane	 81
3.1 Functions of matrices	81
3.2 The exponential of a matrix	83
3.3 Computing the exponential of a matrix	89
3.4 Differential equations and phase portraits	95
3.5 Applications of differential equations	103
Summary	113
Exercises	113
 4 Scalar products	 120
4.1 The euclidean scalar product	120

4.2	The Gram–Schmidt process	124
4.3	Quadratic forms and symmetric matrices	131
4.4	Normal modes	137
4.5	Normal modes in higher dimensions	141
4.6	Special relativity	148
4.7	The Poincaré group and the Galilean group	157
4.8	Momentum, energy and mass	160
4.9	Antisymmetric forms	166
	Summary	167
	Exercises	168
5	Calculus in the plane	175
	Introduction	175
5.1	Big ‘oh’ and little ‘oh’	178
5.2	The differential calculus	183
5.3	More examples of the chain rule	189
5.4	Partial derivatives and differential forms	197
5.5	Directional derivatives	205
5.6	The pullback notation	209
	Summary	214
	Exercises	215
6	Theorems of the differential calculus	219
6.1	The mean-value theorem	219
6.2	Higher derivatives and Taylor’s formula	222
6.3	The inverse function theorem	230
6.4	Behavior near a critical point	240
	Summary	242
	Exercises	242
7	Differential forms and line integrals	247
	Introduction	247
7.1	Paths and line integrals	250
7.2	Arc length	264
	Summary	266
	Exercises	267
8	Double integrals	272
8.1	Exterior derivative	272
8.2	Two-forms	277
8.3	Integrating two-forms	278
8.4	Orientation	285
8.5	Pullback and integration for two-forms	289
8.6	Two-forms in three-space	295
8.7	The difference between two-forms and densities	297
8.8	Green’s theorem in the plane	298
	Summary	305
	Exercises	306
9	Gaussian optics	311
9.1	Theories of optics	311

Cambridge University Press

978-0-521-40649-9 - A Course in Mathematics for Students of Physics: 1

Paul Bamberg and Shlomo Sternberg

Table of Contents

[More information](#)*Contents of Volume 1*

vii

9.2	Matrix methods	315
9.3	Hamilton's method in Gaussian optics	324
9.4	Fermat's principle	326
9.5	From Gaussian optics to linear optics	328
	Summary	335
	Exercises	335
10	Vector spaces and linear transformations	340
	Introduction	340
10.1	Properties of vector spaces	341
10.2	The dual space	342
10.3	Subspaces	343
10.4	Dimension and basis	345
10.5	The dual basis	350
10.6	Quotient spaces	352
10.7	Linear transformations	358
10.8	Row reduction	360
10.9	The constant rank theorem	368
10.10	The adjoint transformation	374
	Summary	377
	Exercises	378
11	Determinants	388
	Introduction	388
11.1	Axioms for determinants	389
11.2	The multiplication law and other consequences of the axioms	396
11.3	The existence of determinants	397
	Summary	399
	Exercises	399
	Further reading	401
	Index	404

 CONTENTS OF VOLUME 2

12	The theory of electrical networks	407
	Introduction	407
12.1	Linear resistive circuits	411
12.2	The topology of one-dimensional complexes	419
12.3	Cochains and the d operator	429
12.4	Bases and dual bases	431
12.5	The Maxwell methods	433
12.6	Matrix expressions for the operators	436
12.7	Kirchhoff's theorem	444
12.8	Steady-state circuits and filters	446
	Summary	451
	Exercises	451
13	The method of orthogonal projection	458
13.1	Weyl's method of orthogonal projection	458
13.2	Kirchhoff's method	461
13.3	Green's reciprocity theorem	466
13.4	Capacitive networks	469
13.5	Boundary-value problems	474
13.6	Solution of the boundary-value problem by Weyl's method of orthogonal projection	477
13.7	Green's functions	482
13.8	The Poisson kernel and random walk	485
13.9	Green's reciprocity theorem in electrostatics	487
	Summary	492
	Exercises	492
14	High dimensional complexes	502
14.1	Introductory remarks	502
14.2	Dual spaces and cohomology	520
	Summary	526
	Exercises	526
15	Complexes situated in \mathbb{R}^n	532
	Introduction	532
15.1	Exterior algebra	535
15.2	k -forms and the d operator	539
15.3	Integration of k -forms	541

Cambridge University Press

978-0-521-40649-9 - A Course in Mathematics for Students of Physics: 1

Paul Bamberg and Shlomo Sternberg

Table of Contents

[More information](#)*Contents of Volume 2*

ix

15.4	Stokes theorem	553
15.5	Differential forms and cohomology	564
	Summary	574
	Exercises	574
16	Electrostatics in \mathbb{R}^3	583
16.1	From the discrete to the continuous	583
16.2	The boundary operator	585
16.3	Solid angle	586
16.4	Electric field strength and dielectric displacement	588
16.5	The dielectric coefficient	596
16.6	The star operator in Euclidean three-dimensional space	597
16.7	Green's formulas	600
16.8	Harmonic functions	602
16.9	The method of orthogonal projection	604
16.10	Green's functions	606
16.11	The Poisson integral formula	608
	Summary	612
	Exercises	612
17	Currents, flows and magnetostatics	615
17.1	Currents	615
17.2	Flows and vector fields	616
17.3	The interior product	621
17.4	Lie derivatives	626
17.5	Magnetism	628
	Appendix: an alternative proof of the fundamental formula of differential calculus	633
	Summary	636
	Exercises	636
18	The star operator	638
18.1	Scalar products and exterior algebra	638
18.2	The star operator	641
18.3	The Dirichlet integral and the Laplacian	646
18.4	The \square operator in spacetime	651
18.5	The Clifford algebra	653
18.6	The star operator and geometry	660
18.7	The star operator and vector calculus	662
	Appendix: tensor products	664
	Summary	674
	Exercises	674
19	Maxwell's equations	686
19.1	The equations	686
19.2	The homogeneous wave equation in one dimension	689
19.3	The homogeneous wave equation in \mathbb{R}^3	692
19.4	The inhomogeneous wave equation in \mathbb{R}^3	695
19.5	The electromagnetic Lagrangian and the energy– momentum tensor	697
19.6	Wave forms and Huyghens' principle	700
	Summary	704
	Exercises	704

Cambridge University Press

978-0-521-40649-9 - A Course in Mathematics for Students of Physics: 1

Paul Bamberg and Shlomo Sternberg

Table of Contents

[More information](#)

x

Contents of Volume 2

20	Complex analysis	706
	Introduction	706
20.1	Complex-valued functions	707
20.2	Complex-valued differential forms	709
20.3	Holomorphic functions	711
20.4	The calculus of residues	715
20.5	Applications and consequences	724
20.6	The local mapping	729
20.7	Contour integrals	735
20.8	Limits and series	740
	Summary	744
	Exercises	744
21	Asymptotic evaluation of integrals	750
	Introduction	750
21.1	Laplace's method	750
21.2	The method of stationary phase	755
21.3	Gaussian integrals	758
21.4	Group velocity	761
21.5	The Fourier inversion formula	762
21.6	Asymptotic evaluation of Helmholtz' formula	764
	Summary	766
	Exercises	766
22	Thermodynamics	768
22.1	Carathéodory's theorem	769
22.2	Classical thermodynamics according to Born and Cathéodory	775
22.3	Entropy and absolute temperature	780
22.4	Systems with one configurational variable	785
22.5	Conditions for equilibrium	796
22.6	Systems and states in statistical mechanics	800
22.7	Products and images	805
22.8	Observables, expectations and internal energy	808
22.9	Entropy	814
22.10	Equilibrium statistical mechanics	816
22.11	Quantum and classical gases	823
22.12	Determinants and traces	826
22.13	Quantum states and quantum logic	831
	Summary	835
	Exercises	836
	Appendix	838
	Further reading	845
	Index	848