

---

## INDEX

---

- absolute double integrals 281
- absolute line integral 265
- addition of velocities, law of 155
- adjoint transformation 374–7
- affine coordinate function 15
- affine geometry 18
- affine plane 1
- affine space, axioms for 9
- affine transformation 17
- affine transformations as  $3 \times 3$  matrices 50
- annihilator 356
- arc length 265
- areas and determinants 26
  
- basis 39, 348
- basis, change of 41
- beats 141
- big “oh”, little “oh” 178
- bijjective 14
- bilinear 123, 272
- Born expansion 194
  
- Cartan xiii
- Cayley–Hamilton theorem 61
- chain rule 184
- change of basis matrix 41, 349
- characteristic equation 59
- characteristic polynomial 59
- closed forms 262
- cofactor matrix 398
- collision 161
- column reduction 367
- composition 15
- conditional probability 67
- conformal linear transformation 57
- conjugate planes 318
- conservation of energy 161
- conservation of energy momentum 165
- conservation of momentum 161
- constant rank theorem 368
- coordinate function 15
- coupled oscillators 138
- critically damped oscillator 105
  
- De Moivre’s theorem 58
- determinant 30, 388–99
- determinant, axioms for 389–390
- differentiable 180
- differential form, linear 199, 247–61
- differential of a map 180
- dimension 347
- directional derivatives 205–9
- dual basis 350–2
- dual space 342–3
  
- Ehrenfest model 78
- eigenvalue 59
- eigenvector 59
- eikonal 324
- elastic collision 161
- energy momentum vector 165
- Euclidean scalar product 120–4
- Euclidean transformation 16
- exact forms 250
- exponential of a matrix 83
- exponential series 82
- extending a basis 349
- exterior derivative 275, 276, 305
- exterior product 275
  
- Fermat’s principle 326–8
- Fibonacci sequence 80
- focal length 319
- force field 248
- forced oscillator 108
- formal power series 82
- forward region 151
- fundamental theorem of affine geometry 43
- fundamental theorem of projective geometry 54
  
- Galilean transformation 160
- Gauss decomposition 322
- Gibbs xii
- Gram–Schmidt process 124–131
- Grassmann xiii
- Green’s theorem 298–303

- Hessian** 225  
**image** 37, 358  
**image, finding a basis of** 367  
**implicit function theorem** 238  
**inelastic collision** 161  
**injective** 13  
**inverse function theorem** 230–7  
**inverse of a matrix** 32, 33  
**isomorphism** 40  
**Kepler motion** 195  
**kernel** 37, 358  
**kernel, finding a basis of** 367  
**Lagrange multipliers** 227–9  
**Laplace's equation** 227  
**Laplacian** 227  
**law of cosines** 124  
**light cone** 152  
**line integrals** 250–64  
**linear dependence** 7, 34  
**linear differential form** 199, 247–61  
**linear independence** 7, 34, 53  
**linear optics** 328–35  
**linear transformation** 18, 20  
**lines, parametrization of** 3, 4  
**Lorentz transformation** 152  
**map** 13  
**Markov process** 66  
**mass** 161–3  
**matrix addition** 24  
**matrix multiplication** 22, 25, 26  
**matrix of a linear transformation** 21  
**matrix of a rotation** 22  
**mean value theorem** 219–222  
**momentum in Newtonian mechanics** 163  
**momentum in special relativity** 165  
**Morse index theorem** 328  
**Newton's method** 231  
**nilpotent matrix** 38, 39  
**non-singular** 20  
**normal forms for matrices** 60, 62, 63  
**normal modes** 137–48  
**normal modes as waves** 145  
**null cone** 152  
**one-dimensional vector space** 10  
**optical length** 325  
**orientation** 31, 285–9  
**orthogonal projection** 130  
**orthonormal basis** 127  
**oscillator** 103–12  
**overdamped oscillator** 106  
**perspective** 51  
**phase portrait** 95–103  
**phase shift** 112  
**Picard's method** 233  
**piecewise differentiable path** 251  
**Poincaré transformation** 154  
**point characteristic** 324  
**positive definite** 123  
**principal planes** 322  
**probability** 66, 67  
**projection** 38  
**projective plane** 53  
**proper Lorentz transformation** 152  
**pullback** 209–13, 289–95  
**Quadratic form** 133  
**Quotient space** 354  
**rank–nullity theorem** 358  
**regular** 20  
**resonance** 112, 141  
**response curve** 112  
**rest mass** 165  
**reverse triangle inequality** 157  
**Riemann** xv  
**row reduction** 360–8  
**saddle point** 135  
**scalar product, axioms for** 123, 131  
**simultaneity** 154  
**singular linear transformation** 20  
**Snell's law** 313  
**Snell's law, linearized version** 317, 318  
**solution set theorem** 371  
**spacetime** 149  
**special relativity** 148–66  
**star shaped** 261–2  
**steady state solution** 110  
**stochastic matrix** 71  
**subspace** 343–4  
**surjective** 14  
**symmetric matrix** 133  
**symplectic group** 167  
**symplectic linear transformation** 167  
**symplectic scalar product** 167  
**tangent space** 208  
**Taylor's formula** 222–5  
**thin lens** 318  
**thin lens, matrix of** 319  
**time, Newton's concept of** 11–13  
**trace of a matrix** 38  
**transition probability** 68  
**translation** 18  
**transpose** 133  
**twin paradox** 157  
**two forms in  $\mathbb{R}^2$**  277  
**two forms in  $\mathbb{R}^3$**  296  
**undamped oscillator** 104  
**underdamped oscillator** 105  
**variation of parameters** 109  
**vector space** 7, 341–2  
**vector space, axioms for** 8  
**velocity transformation** 160  
**wedge product** 275