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978-0-521-40446-4 - Intersection and Decomposition Algorithms for Planar Arrangements

Pankaj K. Agarwal

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This book presents a study of various problems related to arrangements of lines, segments, or curves in the plane. It starts with a discussion of Davenport-Schinzel sequences, which have been applied to obtain optimal or almost-optimal bounds for numerous combinatorial as well as algorithmic problems involving arrangements. The main result here establishes almost tight bounds on the maximal length of (n,s) -Davenport-Schinzel sequences.

The second problem studied is: given a collection of “red” Jordan arcs and another of “blue” arcs, determine whether any red arc intersects any blue arc. Some fast deterministic algorithms are presented, along with applications to many other problems, including collision detection.

Next, a partitioning algorithm is presented that improves the time complexity of a variety of problems involving lines or segments in the plane. Several applications are discussed, most importantly a fast deterministic algorithm to construct a family of spanning trees with low stabbing number. This is shown to be a versatile tool that provides efficient algorithms for a variety of query-type problems, including ray shooting in an arrangement of segments, polygon containment and implicit hidden surface removal.

Researchers in computational and combinatorial geometry should find much to interest them in this book.

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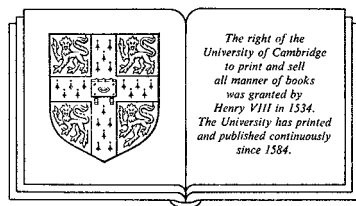
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Intersection and Decomposition Algorithms for Planar Arrangements

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PREFACE

Computational geometry emerged as a scientific discipline about fifteen years back. It has developed very rapidly during the last few years because of its applications in areas such as robotics, computer graphics, VLSI, CAD, solid modeling, etc. Although it primarily concerns designing efficient algorithms for geometric problems, the algorithmic and combinatorial questions are so intertwined in many problems that it is almost impossible to separate one from another. One such area in computational geometry is *arrangements* of lines or curves in the plane, or of hyperplanes and surfaces in higher dimensions. Recently arrangements have received a considerable amount of attention because several fundamental geometric problems can be formulated in terms of arrangements.

This book, based on the author's PhD thesis, studies several algorithmic as well as combinatorial problems involving arrangements of arcs in the plane. The first problem studied here concerns obtaining sharp bounds on the maximum length of *Davenport-Schinzel sequences*. These sequences have been applied to numerous combinatorial as well as algorithmic problems involving arrangements of arcs.

In Chapter 3 we study the *red-blue intersection* problem and its applications to other problems, including collision detection.

Chapter 4 and 5 discuss the following decomposition problem: Given a set of n lines in the plane and a parameter $r \leq n$, decompose the plane into $O(r^2)$ triangles, each of which intersects at most n/r original lines. Chapter 4 presents an efficient algorithm for computing such a partitioning. The application of this algorithm to various other problems is the topic of Chapter 5. The significance of this partitioning algorithm is evident from the problems discussed in Chapter 5.

Finally, in Chapter 6 we show that a spanning tree of a set of points in the plane with the property that each line intersects only few edges of the tree is a versatile tool and that it provides fast procedures for a variety of query-type problems including ray shooting and implicit hidden surface removal.

The open problems mentioned at the end of each chapter suggest further research problems in these areas.

I would like to express my deep sense of gratitude to my thesis advisor

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Micha Sharir. It is difficult to imagine how I would have completed this work without his guidance, encouragement and unwavering support. I would like to thank him for reading various versions of the manuscript. Thanks are also due to Boris Aronov, Bernard Chazelle, Jiří Matoušek, Ricky Pollack, and two anonymous referees for several useful comments, and to Lauren Cowles for a careful reading of the manuscript.

Pankaj K. Agarwal

Contents

1	Introduction	1
1.1	Arrangements	2
1.2	Davenport-Schinzel Sequences	8
1.3	Random Sampling and Deterministic Partitioning	11
1.4	Spanning Trees with Low Stabbing Number	14
2	Davenport-Schinzel Sequences	17
2.1	Introduction	17
2.2	The Upper Bound for $\lambda_4(n)$	20
2.2.1	Decomposition of DS-sequences into chains	20
2.2.2	Properties of Ackermann's and related functions	22
2.2.3	Upper bound for $\Psi_4(m, n)$	28
2.3	The Lower Bound for $\lambda_4(n)$	34
2.3.1	The functions $F_k(m)$ and their properties	34
2.3.2	The sequence $S_k(m)$	37
2.4	The Upper Bounds for $\lambda_s(n)$	42
2.4.1	Upper bounds for $\Psi_s(m, n)$	44
2.5	The Lower Bounds for $\lambda_s(n)$	54
2.5.1	The functions $F_k^s(m)$, $N_k^s(m)$, $F_k^\omega(m)$ and their properties	55
2.5.2	The sequences $S_k^s(m)$	62
3	Red-blue Intersection Detection Algorithms	69

3.1	Introduction	69
3.2	Intersection between a Simple Polygon and Arcs	71
3.3	Applications to Collision Detection and Motion Planning	76
3.3.1	The algorithm	80
3.3.2	Applications to motion planning	82
3.4	Red-blue Intersection Detection in General	88
3.4.1	Deterministic algorithm	89
3.4.2	Relation between many faces and red-blue intersection detection	96
3.5	Further Applications	99
3.6	Discussion and Open Problems	103
4	Partitioning Arrangements of Lines	105
4.1	Introduction	105
4.2	Geometric Preliminaries	109
4.3	Selecting the κ^{th} Leftmost Intersection Point	113
4.3.1	Counting intersection points inside a convex quadri- lateral	113
4.3.2	Computing the κ^{th} leftmost intersection	114
4.4	Partitioning a Convex Quadrilateral	116
4.4.1	A special case	117
4.4.2	General algorithm	122
4.4.3	Analysis of the algorithm	126
4.5	Partitioning the Plane into Quadrilaterals	135
4.6	Constructing Approximate Levels	139
4.7	Coping with Degenerate Cases	146
4.8	Discussion and Open Problems	150
5	Applications of the Partitioning Algorithm	153
5.1	Introduction	153
5.2	Computing or Detecting Incidences between Points and Lines	157
5.3	Computing Many Faces in Arrangements of Lines	160

5.4	Computing Many Faces in Arrangements of Segments	167
5.5	Counting Segment Intersections	173
5.6	Counting and Reporting Red-blue Intersections	176
5.7	Batched Implicit Point Location	181
5.7.1	Polygon containment problem — batched version	185
5.7.2	Implicit hidden surface removal — batched version	186
5.8	Approximate Half-plane Range Queries	188
5.9	Computing Spanning Trees with Low Stabbing Number	189
5.10	Space Query-time Tradeoff in Triangle Range Search	197
5.11	Overlapping Planar Maps	204
5.12	Discussion and Open Problems	209
6	Spanning Trees with Low Stabbing Number	211
6.1	Introduction	211
6.2	Spanning Trees of Low Stabbing Number	214
6.3	Ray Shooting among Non-intersecting Segments	216
6.4	Tradeoff between Space and Query Time	224
6.4.1	The case of quadratic storage	225
6.4.2	The general case	226
6.4.3	Coping with degenerate cases	230
6.5	Reporting All Intersections	232
6.6	Ray Shooting in General Arrangements of Segments	234
6.6.1	Preprocessing the segments	235
6.6.2	Answering a query	238
6.6.3	Analysis of the algorithm	239
6.6.4	Tradeoff between space and query time	243
6.7	Implicit Point Location — Preprocessing Version	247
6.8	Other Applications	249
6.8.1	Polygon containment problem — preprocessing version	250
6.8.2	Implicit hidden surface removal — preprocessing version	254
6.8.3	Polygon placement problem	255

Cambridge University Press

978-0-521-40446-4 - Intersection and Decomposition Algorithms for Planar Arrangements

Pankaj K. Agarwal

Frontmatter

[More information](#)

xiv

6.9 Discussion and Open Problems	256
Bibliography	259
Index of Symbols	273
Index of Keywords	275

List of Figures

1.1	An arrangement of lines	3
1.2	A non-convex face in an arrangement of segments	4
1.3	Lower envelope of a set of curves	6
3.1	Arcs intersecting a polygon	72
3.2	Illustration for Lemma 3.7	75
3.3	Area swept by a wall	79
3.4	Right angle corridor	84
3.5	General corridors	86
3.6	Illustration for Lemma 3.18	91
3.7	Illustration for Lemma 3.24	97
3.8	Detecting intersection between billiards balls	100
4.1	Exact, approximate and simplified levels	109
4.2	Lines intersecting a convex quadrilateral	111
4.3	A segment and its dual	112
4.4	A green line and its endpoints	114
4.5	Partitioning \mathcal{Q} into subquadrilaterals	118
4.6	Cells intersecting a line	120
4.7	Red, green and blue pseudo edges	123
4.8	Internal and boundary cells	124
4.9	Cells with seven or eight edges	125
4.10	Charging of a red-green intersection point	127
4.11	Structure of a boundary cell	128

4.12	Illustration for Lemma 4.17 and Lemma 4.18	130
4.13	Charging of line cell crossings	131
4.14	A triangle intersecting two levels $\frac{\pi}{r}$ apart	140
4.15	Illustration for Lemma 4.30	142
4.16	Triangulation induced by approximate levels	144
4.17	Illustration of Lemma 4.35	145
4.18	Cells in degenerate cases	148
5.1	Incidence counting problem	157
5.2	A face in an arrangement of lines, and its dual	161
5.3	An arrangement and its dual graph	162
5.4	Constructing a spanning path	163
5.5	Spanning tree and a spanning path of a set of points	163
5.6	Zone of a triangle	166
5.7	Binary tree formed on triangles	170
5.8	A long segment and its endpoints	174
5.9	Intersections between long segments	178
5.10	Spanning path of a set of points	191
5.11	Maximal matching of endpoints	192
5.12	A triangle and its dual	198
5.13	Slanted range searching	200
5.14	Two types of triangles	203
5.15	Long blue segments	206
5.16	Planar map formed by long blue segments	207
6.1	Ray shooting in arrangements of segments	212
6.2	Binary tree formed on a spanning path	215
6.3	Spanning paths on endpoints of segments	217
6.4	Two sided segments	217
6.5	Two different orderings of a pair of segments	219
6.6	Illustration for Lemma 6.7	220
6.7	Segments e_h , e_u and e_{ϕ_p}	221

Cambridge University Press

978-0-521-40446-4 - Intersection and Decomposition Algorithms for Planar Arrangements

Pankaj K. Agarwal

Frontmatter

[More information](#)

xvii

6.8	Convex hull of right endpoints of a set of segments	223
6.9	Handling degenerate cases	231
6.10	Partitioning segments into different subsets	235
6.11	Zone of a triangle for a set of segments	236
6.12	Zone of a triangle for long segments	237
6.13	Polygon formed by the zone of long segments	239
6.14	Polygon containment problem	250
6.15	Dividing segments into two subsets	252