This book outlines the basic science underlying the prediction of stress and velocity distributions in granular materials. It takes the form of a textbook suitable for post-graduate courses, research workers and for use in design offices. The nature of a rigid-plastic material is discussed and a comparison is made between the Coulomb and Conical (extended von Mises) models. The methods of measuring material properties are described and an interpretation of the experimental results is considered in the context of the Critical State Theory.

The early chapters consider the traditional methods for predicting the forces on planar retaining walls and the walls of bunkers and hoppers. These approximate methods are described and their accuracy discussed. Later chapters give details of the exact methods of stress and velocity prediction, covering both the radial stress and velocity fields and the method of characteristics. The analysis of stress and velocity discontinuities is also considered as is the prediction of the mass/core flow transition. The final chapter covers the discharge rate of materials through orifices, dealing with both the correlations of experimental results and theoretical prediction. The influence of interstitial pressure gradients is also considered leading to an analysis of the flow of fine materials and the effects of air-augmentation. The book ends with an assessment of Jenike's method for predicting the circumstances under which cohesive arching prevents flow.

The book will be an invaluable text for all those working with or doing research into granular materials. Exercises and solutions are provided which will be particularly useful for the student.

STATICS AND KINEMATICS OF GRANULAR MATERIALS

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CAMBRIDGE
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, Victoria 3166, Australia

First published 1992

A catalogue record for this book is available from the British Library

Library of Congress cataloguing in publication data

Nedderman, R. M.
Statics and kinematics of granular materials / R. M. Nedderman.
p. cm.
Includes bibliographical references
ISBN 0-521-40435-5 (hardback)
1. Granular materials. 2. Strains and stresses. 3. Kinematics.
I. Title.
TA418.78.N44 1992
620.1′9–dc20 91-39970 CIP

ISBN 0 521 40435 5 hardback

Transferred to digital printing 2004

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Notation

Note on subscripts

The co-ordinate variables, x, y, z, r, θ, χ are used as subscripts to denote the appropriate component of the parameter. Stresses have two subscripts, the first denoting the plane on which the stress acts and the second, the direction in which the associated force acts. The subscript w denotes conditions at the wall.
The subscripts A and P denote the active and passive cases.

The overbar denotes an average quantity.
Superscripts $^+$ and $^-$ denote conditions on either side of a discontinuity.

The list of symbols below gives only those parameters which appear on several occasions.
Symbols that appear only transiently and are defined in the text are not included.

a	Direction cosine (appendix 1); distance
b	Horizontal distance; breadth
c	Cohesion; parameter defined by equation (5.4.19)
c_e	Equivalent cohesion defined by equation (6.5.9)
d	Particle diameter
e	Centre of Mohr's strain rate circle; voids ratio as defined by equation (6.2.3)
$f(\,)$	Function
f_c	Unconfined yield strength
ff	Flow factor, defined by equation (10.8.6)
g	Acceleration due to gravity
h	Height; effective height of cone (§5.7)
h_c	Depth of tension cracking

k	Ratio μ_w/μ (chapter 4); parameter in Rosin–Rammler distribution (§6.3); permeability defined by equation (6.4.1); parameter in the Beverloo correlation, equation (10.2.4)
l	Length
m	Mass
m	Parameter defined by equation (5.8.10)
m^*	Parameter defined by equation (5.8.21)
m^{**}	Parameter defined by equation (5.8.28)
n	Parameter in Warren Spring equation (6.5.8); co-ordinate normal to a surface
p	Pressure; mean of the major and minor principal stresses
p^*	Distance of the centre of Mohr's circle from the point of intersection of the Coulomb line with the σ axis ($p^* = p + c \cot \phi$)
p_c	Value of p during consolidation
p_i	Isotropic pressure
p_m	Size distribution function by mass (§6.3)
P_n	Size distribution function by number (§6.3)
p_0	Reference pressure; value of p on the centre-line
q	Radius of Mohr's stress circle (§6.2); radial stress field parameter, $p/\gamma r$ (chapters 7 & 8)
q_0	Dimensionless surcharge, $Q_0/\gamma b$
r	Radial co-ordinate; radius
r^*	Dimensionless radial co-ordinate, $2r/D$
r_0	Radius of the free-fall arch
s	Co-ordinate along a surface; deviatoric stress (chapter 9)
t	Tan α (chapter 4)
u	Velocity in the x-direction; Cartesian co-ordinate (appendix 1)
u^*	Change in normal velocity across a discontinuity
v	Velocity in the y-direction; Cartesian co-ordinate (appendix 1)
v^*	Change in shear velocity across a discontinuity
v_i	Interstitial velocity
v_r, v_θ etc.	Velocities in appropriate co-ordinate directions
v_R	Relative velocity
v_s	Specific volume (§6.2)
v_0	Reference specific volume (§6.2)
w	Velocity in the z-direction
x	Cartesian co-ordinate; parameter in log-normal distribution (§6.3)
y	Cartesian co-ordinate; parameter in log-normal distri-

	bution (§6.3)
z	Cartesian co-ordinate; distance down far wall (§4.8); parameter in Rosin–Rammler distribution (§6.3)
A	Area; parameter defined by equation (7.4.6) or equation (9.4.4); angle defined by equation (7.9.5)
B	Parameter defined by equation (5.8.8); parameter defined by equation (7.4.7) or equation (9.4.5)
C	Parameter in Warren Spring equation (6.5.8); parameter defined by equation (7.4.8) or equation (7.4.28) or equation (9.4.6); parameter in the Beverloo correlation, equation (10.2.2)
D	Diameter; parameter defined by equation (7.4.9) or equation (9.4.7)
D_0	Orifice diameter
D_a	Arithmetic mean diameter, defined by equation (6.3.31)
D_{max}	Parameter in Avrami distribution, equation (6.3.16)
D_g	Geometric mean diameter
D_{gm}	Geometric mean diameter by mass
D_{H}	Hydraulic mean diameter defined by equation (5.5.6)
D_s	Surface mean diameter, defined by equation (6.3.32)
D_v	Volume mean diameter, defined by equation (6.3.33)
D_{vs}	Volume/surface mean diameter, defined by equation (6.3.34)
E	Parameter defined by equation (7.4.10) or equation (9.4.8)
E_1	Parameter in Ergun equation, defined by equation (6.4.7)
E_2	Parameter in Ergun equation, defined by equation (6.4.8)
E_α	Parameter defined by equation (7.11.9)
E_β	Parameter defined by equation (7.11.10)
F	Force; parameter defined by equation (7.4.11) or equation (7.4.29) or equation (9.4.9)
FF	Flow function (§6.6)
$F(x,\phi_d)$	Function in the Rose and Tanaka correlation, equation (10.2.5)
$F(\theta)$	Velocity function defined by equation (8.4.12)
G	Shear modulus; plastic potential (§8.3); mass flow rate of gas (chapter 10)
H	Dimensionless depth, h/b
I_1,I_2,I_3	Stress invariants (§6.2)
J	Force on far wall
K	Rankine's coefficient of earth pressure (Janssen's constant) (chapters 3 & 5)
K_n,K_n'	Normalisation factors (§6.3)

L	Rotation matrix
M	Mass; parameter defined by equation (9.2.6); momentum flux (chapter 10)
M_c	Value of M in a compression test
M_e	Value of M in an extension test
N	Normal force; number of particles (§6.3)
P	Force; perimeter (§5.5); dimensionless form of p, $p/\gamma a$
P_0	Value of P on the axis of symmetry
P_m	Cumulative size distribution by mass
$P^{l}/8_n$	Cumulative size distribution by number
Q	Surcharge; volumetric flow rate
Q^*	Volumetric flow rate per unit width
Q_0	Uniform surcharge
R	Radius of Mohr's circle; radius; radius of Enstad element (§5.10); dimensionless radial co-ordinate, r/a
R'	Radius of Mohr's strain rate circle
Re	Reynolds number, defined by equation (6.4.10)
R_H	Hydraulic mean radius, defined by equation (5.5.12)
T	Ultimate tensile strength, defined by equation (6.5.8); total energy content, (§10.5)
U	Velocity; superficial velocity
U_i	Interstitial velocity
V	Velocity; volume; specific volume (§6.2); parameter in equation (10.4.3)
V_p	Velocity in the plug flow region
W	Weight; mass flow rate
W_B	Mass flow rate predicted
W_0	Mass flow rate in the absence of air-augmentation
X	Force; Enstad parameter, defined by equation (5.10.13); dimensionless distance x/a
Y	Enstad parameter, defined by equation (5.10.14); dimensionless distance y/a; yield function (§8.3)
Z	Dimensionless depth, z/b
α	Angle; hopper half-angle; parameter in equation (6.6.9)
β	Angle of inclination of the top surface; angle defined by equation (5.10.1); compressibility factor (§6.2)
γ	Shear strain (§3.2); weight density $\rho_b g$
δ	Effective angle of internal friction (§6.6); small quantity; angle defined in figure 7.28
δ_{ij}	Kroenecke delta function
ε	Void fraction; angle defined by equation (3.3.4)
ε_V	Volumetric strain

$\dot{\varepsilon}$	Direct strain rate
$\dot{\varepsilon}_V$	Volumetric strain rate
η	Angle between the wall and the vertical
θ	Angle; angular co-ordinate
ζ	Angle measured anticlockwise from the *x*-direction to the characteristic direction; angle measured anticlockwise from the *x*-plane to the plane of the discontinuity
κ	Parameter taking the value -1 in active failure and $+1$ in passive failure; parameter defined by equation (10.5.23)
λ	Angle defined by equation (2.3.10); compressibility parameter (chapters 6 & 10); plasticity parameter (chapters 8 & 9)
λ_S	Shape factor (§6.3)
μ	Coefficient of internal friction
μ_e	Equivalent coefficient of internal friction, defined by equation (6.5.9)
μ_f	Viscosity of fluid
μ_g	Viscosity of gas
ν	Angle of dilation
ξ	Angle measured anticlockwise from the plane of the discontinuity to the major principal plane (chapter 7); angle defined by equation (9.3.10)
ρ	Density
ρ_b	Bulk density
ρ_f	Fluid density
ρ_g	Gas density
ρ_s	Solid density
σ	Normal stress; standard deviation (§6.3)
$\sigma_1,\sigma_2,\sigma_3$	Principal stresses
σ_c	Normal stress under consolidation conditions (§6.5)
σ_{oct}	Octahedral stress, defined by equation (9.2.3)
σ_R	Reduced normal stress, σ/σ_c
σ_y	Yield stress
τ	Shear stress
τ_c	Shear stress under consolidation conditions
τ_R	Reduced shear stress, τ/σ_c
ϕ	Angle of internal friction
ϕ_c	Angle of friction measured in a compression test
ϕ_e	Equivalent angle of internal friction defined by equation (6.5.9)
ϕ_{e}	Angle of friction measured in an extension test (chapter 8)
ϕ_w	Angle of wall friction

ψ	Angle measured anticlockwise from the x-direction (or the r-direction of cylindrical co-ordinates) to the major principal stress direction; stream function (§8.2)
ψ^*	Angle measured anticlockwise from the r-direction of polar or spherical co-ordinates to the major principal stress direction
χ	Angular co-ordinate
ω	Angle defined by equation (3.7.8)
Δ	Increment; angle of end point locus (§6.6); angle defined by equation (7.12.4)
ΔP	Pressure difference
Λ	Angle defined by equation (2.5.8); compressibility parameter (§6.2)