1
Introduction

A granular material can be defined as any material composed of many individual solid particles, irrespective of the particle size. Thus the term granular material embraces a wide variety of materials from the coarsest colliery rubble to the finest icing sugar. The handling of granular materials is of the greatest importance in the chemical industry, it being estimated that on a weight basis, roughly one-half of the products and at least three-quarters of the raw materials are in the form of granular solids. When one adds to this the vast tonnages of wheat, sugar, iron ore, cement, sand and gravel that have to be stored and transported, the importance of granular materials becomes self-evident. The early dominance of oil in the history of the chemical industry has tended to emphasize the importance of fluid mechanics as one of the main constituents of a chemical engineer's education. It is only relatively recently that the economic advantage of a study of the behaviour of granular materials has been appreciated. One reason for this is that solids handling equipment has long been designed empirically and it is only over the last thirty years that the subject has been placed on a firm numerical basis. By contrast, the quantitative nature of fluid mechanics has been well-known since the work of Darcy in the 1840's.

Granular materials, or bulk solids as they are sometimes called, are usually stored in hoppers or bunkers. These can vary in size from a conventional salt-cellar, containing a few grams, to large installations holding several thousands, or even tens of thousands, of tonnes. The large bunkers storing agricultural products such as grain or sugar can be up to 20 m in diameter and 60 m in height and are a prominent feature of many rural landscapes. The words bunker, hopper, silo and bin tend to be used indiscriminately but it is recommended practice to use the word 'bunker' if the walls are parallel, forming a container of constant cross-section and to use 'hopper' when the walls converge towards a relatively small opening at the base. The words 'bin' and 'silo' are general, covering both bunkers and hoppers, and also covering the frequent case of a cylindrical bunker surmounting a conical hopper as in figure 1.1. The British Code of Practice for silo design, issued by the British Materials Handling Board (BMHB, 1987), at present in draft form, recommends the use of the ugly compound 'silo–bin', but this will not be used here.

As implied in the title, this book is mainly concerned with two aspects of the behaviour of granular materials, both of direct relevance to the problems of storage and transportation of granular materials. The first part of the book, chapters 2 to 7, is concerned with the statics of granular materials. Here the objective is to predict the stress distributions within a granular material with particular emphasis on predicting the forces on the walls of the silo in which the material is stored. The second part deals principally with kinematics, the study of the motion of flowing granular materials. This topic has implications for the prediction of stress distributions, as it is found that frequently there are flowing and stagnant zones within a discharging bunker and that the forces on the walls depend on whether or not the material adjacent to the wall is moving. Finally, in the last chapter we pay attention to the dynamics of granular materials in order to predict the discharge rate from silos.

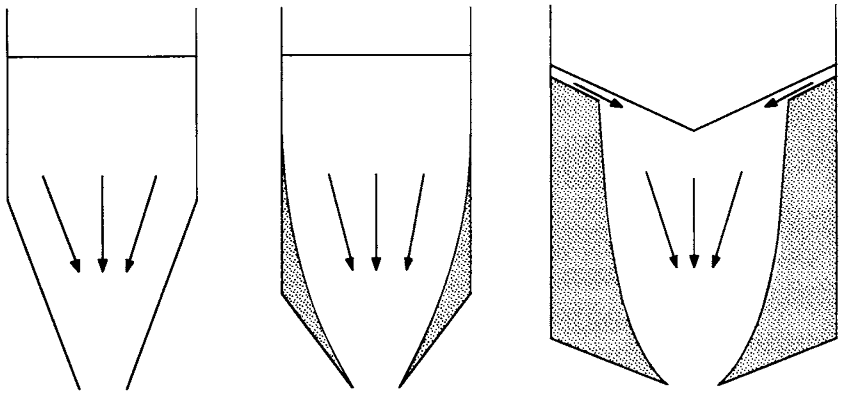

Figure 1.1 Flow patterns in discharging bins.

The analysis of stress and velocity distributions in granular materials is based on principles laid down in the eighteenth century by Coulomb and which have been developed by the soil mechanicists. The emphasis in our case is however different. Whereas in soil mechanics the main objective is to prevent movement of the soil, the converse is true in bulk solids handling especially when considering the discharge rates from silos. Credit must be given to Jenike and his co-workers who in the 1960's first applied the principles of soil mechanics to silo design and developed design procedures that are still in common use. His three famous bulletins and in particular, his third bulletin, take the form of design manuals containing a mixture of rigorous analyses and empirically derived recommended procedures. This style has been followed some 20 years later by Arnold and co-workers (1980) and in the various national codes of practice for hopper design.

The objective of this book is somewhat different. It is intended primarily as an undergraduate or postgraduate textbook in which the basic principles are treated rigorously. The emphasis is on the fundamental science and a careful distinction between established fact and rule-of-thumb is attempted. The subject matter of chapters 1 to 6 and chapter 10 is based on lectures given as part of a course leading to the Master of Engineering (M. Eng.) degree in the Department of Chemical Engineering at Cambridge. The remaining chapters contain material required by research students working on the flow of granular materials. Some of the examples given in the book are based on questions set in Part II of the Chemical Engineering Tripos at Cambridge with kind permission of the Council of the Senate.

The writers of codes of practice and design manuals are obliged to propose some recommendation for all pertinent aspects of silo construction and where no established scientific facts are available, they have to resort to empiricism and propose rules-of-thumb. Unfortunately the temptation to borrow an empirical result from a previous code is often irresistible and in the successive uncritical copying, what started as an admittedly empirical rule-of-thumb, often acquires the status of established fact. One of the objectives of this book is to examine these aspects of traditional wisdom, in an attempt to see which have some scientific foundation. The writer of a textbook does have at least one advantage over the writer of a design manual. When he reaches the limit of his competence he can resort to Aristotle's stratagem and freely admit that he does not know. The silo designer will therefore find this work incomplete but it is hoped that it will enable him to understand the basic principles on which the usual design procedures are based. Though primarily intended for undergraduate and postgraduate use, an attempt has been made to make the book comprehensible on its own so that it can be used by those who do not have the advantage of readily available advice.

In chapter 2 we will establish various fundamental concepts about the analysis of stress and strain. Many readers will already be familiar with these ideas and may wish to pass on directly to chapter 3. However their attention is drawn particularly to §2.3 and §2.5 in which the sign conventions for stress and strain rate are defined, as these differ from those commonly used in fluid mechanics and elasticity. In chapter 3 the nature of the idealised granular material, the so-called Coulomb material, is described and various approximate methods of predicting stress distributions in such materials are outlined in chapters 4 and 5. In chapter 6 we investigate the methods available for measuring the physical properties of a granular material and consider how accurately these conform to the idealised model. Chapter 7 considers the mathematically exact methods of stress analysis and we compare the results with the approximate results of chapters 4 and 5. The analysis of velocity distributions is considered in chapter 8 and this leads on to the consideration of an alternative to the Coulomb model of a granular material which is discussed in chapter 9. Chapters 7 to 9 are, however, mathematically complex and the less mathematically inclined reader might prefer to omit them and pass directly to chapter 10 which deals with discharge rates through orifices.

First, however, we need to define certain terms which will be used frequently throughout the book and discuss some of the more fundamental properties of granular materials.

Perhaps the most important single property of a granular material is the particle size and consequently many authors have sought to classify materials according to their mean particle size. Richards (1966), for example, proposes the classification given in table 1.1. Whilst this is helpful in a qualitative way, it has not been followed slavishly in this book where we will use the phrases 'granular material' and 'bulk solids' indiscriminately to denote all materials composed of many solid particles, irrespective of size.

The measure of particle size must be chosen with care. If the particles are spherical, the diameter is clearly the most convenient dimension but for non-spherical particles some characteristic dimension has to be selected. Frequently this is the equivalent spherical diameter, that is the diameter of the sphere of the same volume, but this is not the

Table 1.1

Particle size range	Name of material	Name of individual component
0.1 μm–1.0 μm	Ultra-fine powder	Ultra-fine particle
1.0 μm–10 μm	Superfine powder	Superfine particle
10 μm–100 μm	Granular powder	Granular particle
100 μm–3.0 mm	Granular solid	Granule
3.0 mm–10 mm	Broken solid	Grain

only possibility and other measures of particle size are discussed in chapter 6. Furthermore, most materials contain a range of particle sizes so that we must also consider the particle size distribution. It is found that the presence of a few fines can have a marked effect on the behaviour of the material, so that the simple classification based on mean particle size alone is an over-simplification. Particle size distributions are also considered in chapter 6.

Though ultimately one expects that we will be able to predict the majority of material properties from the particle size distribution, the subject has not yet developed that degree of understanding. Some considerable progress has been made, notably by Molerus (1982), and Briscoe and Adams (1987) but it is still convenient in many cases to treat a granular material as a continuum and to measure its bulk properties without enquiring in detail about their causation. In particular, both the density and frictional properties are more commonly measured than predicted and in this respect granular materials do not differ from conventional fluids. No doubt in time, we will be able to predict the density and viscosity of a liquid from its molecular properties, but these quantities are, for the present, always obtained experimentally.

There is, however, a particular problem with regard to density, since there are two densities of interest, the density of the particles themselves, which we will call the *solid density* and denote by ρ_s, and the density of the mixture of solid and interstitial gas which is known as the *bulk density* ρ_b. Provided the particles are not porous, the solid density can be measured by the usual techniques of liquid displacement and the bulk density can be obtained from the ratio of the mass and volume of a sample.

These two densities are related by

$$\rho_b = \rho_s(1 - \varepsilon) \qquad (1.1)$$

where ε is the void fraction defined as the volumetric fraction of the material occupied by the interstitial gas. Strictly equation (1.1) should be written

$$\rho_b = \rho_s(1 - \varepsilon) + \rho_g \varepsilon \tag{1.2}$$

where ρ_g is the gas density. However, since the gas density is typically one-thousandth of that of the solid, equation (1.1) is sufficiently accurate.

Whilst the particles themselves may be compressible, the change in solid density over the range of stresses normally encountered is usually small, so that ρ_s is effectively a constant for a given material. On the other hand, the bulk density is found to vary significantly with applied stress, mainly as a result of rearrangement of the particles. Unfortunately on reduction of the stress, the material does not necessarily expand and as a result the bulk density depends not only on the current stress in the material but also on its stress history. Thus for a given material ρ_s may be treated as a constant but the value of ρ_b will depend on the present *and* past treatment of the material.

When considering the flow pattern within a discharging bunker, it is usual to distinguish between mass and core flow. In a mass flow hopper, all the material is in motion as illustrated in figure 1.1(*a*). In such a hopper the first material to be loaded is the first to be discharged, giving the 'first in, first out' flow pattern. However, mass flow can only occur in comparatively narrow hoppers. If the hopper half-angle α is large the flow will be confined to a narrow core surrounded by stagnant material as illustrated in figures 1.1(*b*) and 1.1(*c*). If the core is narrower than the width of the silo, as in figure 1.1(*c*), the material near the top will cascade down the top surface into the flowing core and will be discharged before material at a lower level, giving the 'first in, last out' pattern. However, the width of the flowing core normally increases with height and for a tall, narrow silo the flowing core will reach the upper parts of the walls. The Draft British Code of Practice (BMHB, 1987) subdivides what is usually known as core flow, i.e. the patterns illustrated by figures 1.1(*b*) and 1.1(*c*), into core flow in the strict sense in which the core reaches the upper parts of the walls, as in figure 1.1(*b*), and internal flow in which the flowing core never reaches the wall, as in figure 1.1(*c*). In this work we will use the older definitions of mass flow, in which all the material is moving, and core flow, in which some of the material is stagnant.

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2

The analysis of stress and strain rate

2.1 Introduction

In this chapter we will develop relationships for the analysis of stress and rates of strain which will be familiar to many readers from their knowledge of fluid mechanics or elasticity. Such readers may wish to proceed directly to chapter 3, but their attention is drawn to §2.3 and §2.5 in which the sign conventions used in this book are defined, since these differ from those commonly used in fluid mechanics.

The nature of forces and stresses is discussed in §2.2 and in particular we note that force is a vector but that stress is a somewhat more complicated quantity and cannot therefore be resolved by the familiar techniques of vector resolution. The simplest method for determining the stress components on a particular plane is known as Mohr's circle and this is derived in §2.3 and compared with alternative methods in appendix 1.

Forces are generated as a result of stress gradients and these are related to the acceleration of the material by Euler's equation which is derived in §2.4. Finally in §2.5, we define the strain rate in terms of the velocity gradients and note that strain rates, like stresses, can be analysed by means of a Mohr's circle.

In an attempt to reduce the tedium of this chapter, most of the derivations are presented only for Cartesian co-ordinates and the results for other co-ordinate systems are given, without derivation, in the appendices.

2.2 Force, stress and pressure

It is assumed that the reader is fully familiar with the concept of force and with the fact that force is a vector. As a consequence of its vectorial nature, a force F can be expressed in terms of its components F_x, F_y and F_z parallel to the three co-ordinate directions and by convention these components are taken to be positive when acting in the direction of the co-ordinate increasing. Forces can be resolved in any chosen direction by the techniques of vector algebra but it is more usual to rely on a graphical construction known as the triangle of forces. This construction is, however, so simple that it is often possible to write down the answer by inspection without the necessity of drawing the diagram itself. In particular the component of a force F in a direction inclined to it by the angle θ is $F \cos \theta$, a result sometimes known as the cosine law of vector resolution.

The concept of stress is less familiar and is best illustrated by considering an elementary cuboid with edges parallel to the co-ordinate directions as shown in figure 2.1. It is usual to name the faces of such a cuboid according to the directions of their normals and there are therefore two x-faces as shown in the figure. On each face there may be a force and we will denote that acting on one of the x-faces by F_x. Since the cuboid is of infinitesimal size the force on the other x-face will not differ significantly. The force F_x will not necessarily be normal to the x-face and we can resolve it into its components in the three co-ordinate directions, F_{xx}, F_{xy} and F_{xz}. Dividing by the area of the x-face, A_x, we obtain the stresses on that face and it is usual to distinguish between the normal stress σ_{xx}, obtained from F_{xx}, and the other two stresses which are called shear stresses and denoted by τ_{xy} and τ_{xz}.

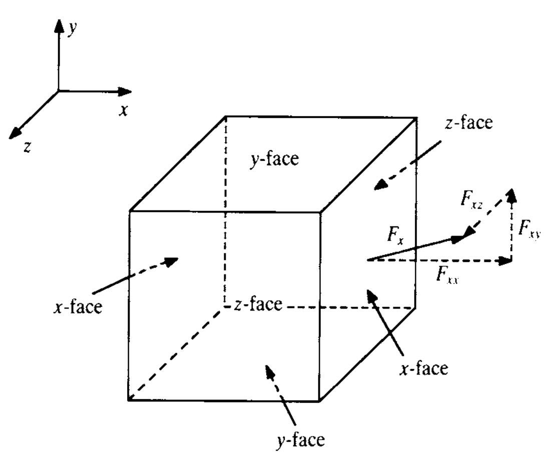

Figure 2.1 Components of force acting on the face of an elementary cuboid.

There are similarly three stress components on each of the two remaining pairs of faces, so that in total we have nine stress components which may be written in the form,

$$\begin{matrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{matrix}$$

It should be noted that in this formulation, the first subscript refers to the face on which the stress acts and the second subscript to the direction in which the associated force acts. A vector has three components in three-dimensional space and it is therefore clear that a stress, having nine components, cannot be a vector.

The components of a stress in any other set of co-ordinate directions can be obtained by matrix manipulations or by the techniques of tensor analysis. Fortunately most of the problems with which we are concerned are essentially two-dimensional; for such systems the very much simpler device known as Mohr's circle can be used and this has particular advantages as it fits conveniently with the basic relationships governing the behaviour of granular materials. Mohr's circle is considered in the next section and the matrix methods of co-ordinate transformation are given in appendix 1.

Circumstances can occur in which all three normal stresses are equal and all the shear stresses are zero. This is more common in the field of fluid mechanics and is known as a state of isotropic pressure. Pressure, which is a scalar since it acts equally in all directions, is therefore a particular case of a stress. Unfortunately, the words 'pressure' and 'stress' tend to be used indiscriminately. For example the Draft British Code of Practice (BMHB, 1987) recommends the use of 'stress' within the material but denotes the stresses exerted on the containing walls as 'pressures'. This usage is contrary to that commonly found in mechanics and will be avoided in this book. We will use the word 'stress' to apply to both internal and external stresses and reserve the word 'pressure' to the cases when the stress state is isotropic or when we need to consider the motion of the interstitial medium, which is usually air.

2.3 Two-dimensional stress analysis – Mohr's circle

Many of the problems of industrial importance have sufficient symmetry, either planar or cylindrical, to make a two-dimensional analysis realistic.

This is a great convenience as the manipulation of stresses in two-dimensional systems is very much easier than in three dimensions. The rather more complicated analysis of stress in three-dimensional systems is outlined in appendix 1.

The method we will use is known as Mohr's circle. This method does, however, have one disadvantage in that it requires a different sign convention from that required for matrix or tensorial manipulation of stresses. Some authors have attempted to combine the sign conventions by unsatisfactory devices such as reversing the signs of shear stresses if the subscripts are in alphabetical order. It is the opinion of the present author that it should be acknowledged that different sign conventions are necessary and that one should keep to the one appropriate for the technique in use.

Since granular materials can only rarely take tension, it is convenient to take compressive stresses as positive and, having selected this convention, the use of Mohr's circle requires that shear stresses should be taken as positive when acting on the element in an anticlockwise direction. Recalling that Mohr's circle is applicable only to two-dimensional situations, we can illustrate our sign convention by means of figure 2.2. The directions in which the stresses acting on the element are numerically positive are shown by the arrows in this figure.

If we take moments about an axis normal to the paper we find that, for stability,

$$\tau_{xy} = -\tau_{yx} \qquad (2.3.1)$$

and thus the shear stresses occur as a complementary pair.

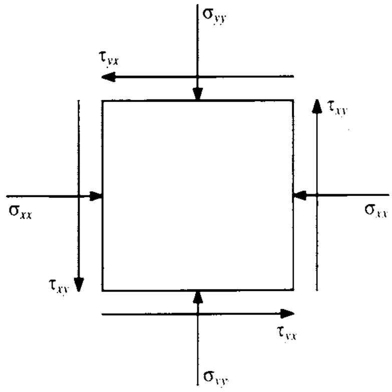

Figure 2.2 Definition of stresses.