Part I

BÉNARD CONVECTION AND RAYLEIGH–BÉNARD CONVECTION

Bénard convection, or its modern offspring, called Rayleigh–Bénard convection, has been the subject of several reviews and has also been discussed in a number of books. The very first review of Bénard convection was written by Pellew and Southwell (1940) and served until 1960 as an excellent summary of the beginnings of the investigation of Bénard convection, in particular of the theoretical aspects of the problem. The first book in which a summary of the Bénard problem was presented was written by Chandrasekhar (1961). His book dealt with hydrodynamic and hydromagnetic stability in general, and Bénard convection and the Taylor vortex instability were described in all the detail known at that time. Chandrasekhar's book appeared when the first studies of nonlinear Bénard convection were being made and when the consequences of surface tension effects had just been outlined. The 44 references at the end of Chapter II of Chandrasekhar's book were a nearly complete list of the publications concerned with Bénard convection up to that time.

After three decades of research on the nonlinear aspects of Bénard convection, the number of relevant publications stands at more than 500, counting only papers whose principal topic is Bénard or Rayleigh–Bénard convection. Only about half of these papers are listed in the references at the end of this book. It is simply impossible and would be distracting to refer to each and every paper that deals with Bénard or Rayleigh–Bénard convection. However I have referred to any additional independent publication which supports a previously reported result, because that is an essential part of verification. I have, instead of going into innumerable detail, chosen to focus attention on what I consider to be the centerpiece of the Bénard and Rayleigh–Bénard convection problem. This is for me the instability of a Boussinesq fluid layer on an infinite plane of excellent thermal conductivity heated uniformly from below and cooled uniformly from above, or since the infinite plane is an abstraction, in containers of large aspect ratio. I stick very closely to the principal problem throughout Chapters 1–7, without pursuing many of the possible nuances which the framework of the principal problem still permits. Since science proceeds from the simple to the complex, the nuances will have to wait until we have reached agreement on the main problems. Only in Chapter 8 are variations of the main theme of the problem discussed; only four of them are examined, although actually there are many more. One of the possible items under miscellaneous topics was convection in porous media. This topic has just been covered in a book by Nield and Bejan (1991). This shows that, if we pursued in detail all topics related to convection, we would have to write a couple of volumes. Of the four variations of the main theme that we cover, two deal with obvious questions resulting from the simplifying assumptions usually made in the theory of convection; the other two variations deal with problems for which linear theories exist, and these theories have been discussed extensively in Chandrasekhar's book. It is, of course, also challenging to discuss topics of very recent interest, for example, binary convection. But I have abstained from discussing binary convection because I lack personal experience in this field and because I believe that independent work remains to be done to verify many of the results presented in this field.

I have focused attention on those papers which contribute most to the clarification of the principal problems. There are, naturally, conflicting views about which papers are convincing and which are not. Care has been taken to provide readers with sufficient sources so that they can sort out things by themselves and form their own opinions. It very quickly becomes evident that it is much easier to write about convincing results. They can be dealt with in much less space than controversial results, which require inordinate explanations because the criticism has to be justified. This is, of course, not the desired order of things, but is in line with the observation that problems of seemingly insurmountable difficulty, once they are solved, afterward appear to be simple. In the references preference has been given to regular journal articles because they are readily available in most university libraries. If, as is now quite common, the results of a particular study have been published piecemeal, as reports, letters, short communications, conference contributions, preliminary results, preprints, etc., only the most comprehensive of these papers has been quoted. Publications which are not in regular journals have been referred to only in exceptional cases. The latest papers referred to are from the printed material available to me at the end of December 1991.

Since 1960 several reviews on convection have been written, as have a number of books in which convection is discussed. The reviews most often used are those of Segel (1966) about early nonlinear theories, of Koschmieder (1974) about early nonlinear experiments, of Palm (1975) about nonlinear theory, of Normand et al. (1977) about a physicist's approach to convection, of Busse (1978) about nonlinear properties of convection on an infinite plane, and of Davis (1987) about thermocapillary instabilities. Books which have chapters on Bénard convection or deal with hydrodynamic stability and refer to convection are those of Saltzman (1962a), which contains a very practical collection of the classical papers on the theory of convection, of Turner (1973) about buoyancy effects in general, of Gershuni and Zhukovitskii (1976) about convection in general, of Joseph (1976) about the application of the energy theory to instability, of Drazin and Reid (1981) about hydrodynamic stability with an extensive discussion of nonlinear stability theory, of Legros and Platten (1983) about the application of numerical techniques to convection, and of Georgescu (1985) about the mathematical aspects of stability theory. Various mathematical and experimental aspects of convection and Taylor vortex flow are discussed in Barenblatt et al. (1983) in the context of the formation of turbulence.

Finally I would like to define the term "convection," which is used so often in this book. Convection in general means fluid motions caused by temperature differences with the temperature gradient pointing in any direction. In Bénard convection and Rayleigh–Bénard convection the temperature differences are applied in the vertical direction. Instead of using the term "convection caused by heating from below" time and again, we shall, for the sake of brevity, just use the word "convection."

1

BÉNARD'S EXPERIMENTS

Convective motions in shallow fluid layers heated from below or cooled from above by evaporation occur naturally under many circumstances. Various kinds of material floating in the fluid frequently make the flow visible. Convection in shallow fluid layers must therefore have been noticed many times before 1900, and was described by several observers. These early observations are only of historical interest because they were not scientific in the sense that they did not take place under controlled conditions and were not made reproducible. It seems that an accidental observation made in 1897 by A. Guébhard, namely the observation of polygonal vortex motions in an abandoned bath of film developer, induced Bénard* to take a thorough look at convection. Bénard then performed the first systematic investigation of convection in a shallow fluid layer heated from below. The results of his studies, actually the results of his doctoral thesis, were published in the paper ''Les tourbillons cellulaires dans une nappe liquide'' (Bénard, 1900) and a subsequent paper (Bénard, 1901). Both papers cover pretty much the same ground. Bénard's work is usually dealt with in the form of a reference; a thorough review of his studies is not available. So, as a matter of fairness, and in order to provide a basis for Chapter 2, we shall describe his experiments here. Several results of his experiments are easy to understand with hindsight. Therefore in the following we shall often assume knowledge of basic concepts of the theoretical explanation of convection.

Bénard went at his experiments methodically. From earlier measurements of the thermal conductivity of fluids he knew that a fluid layer heated from above remains in hydrostatic equilibrium, i.e. at rest. Bénard, on the other hand, set out to study convection by heating from below. He chose to investigate the ''most simple case'' with uniformity of the conditions in the hor-

* Henri Bénard, 1874–1939, professor at the University of Paris.

izontal plane. He realized uniform horizontal conditions in his experiments by reducing as much as possible the influence of the necessarily present lateral wall of the fluid. In other words, Bénard strove to work with an approximation of an infinite horizontal fluid layer. And in order to ensure the uniformity of the temperature at the bottom of the fluid he made the bottom plates out of good thermal conductors, brass or cast iron. The uniformity of the conditions seems to have been of particular importance for him, because he was interested in the consequences of, as we would say, infinitesimal disturbances of the temperature, or in his words, in "très léger excès fortuit et local de température." He wondered whether a center of ascending motion above such a disturbance would remain immobile or whether it would be replaced without regularity. He said that only the experiment could answer this question. By posing this question in the context of an infinite fluid layer he had not only laid the foundation for a successful experiment but also opened the door to a successful theoretical analysis of the problem.

Bénard chose a free surface for the fluid layer, which introduced, as he noted, an asymmetry of the (boundary) conditions, but there was not much choice if he wanted to observe the convective motions in full detail. In his papers there is a noncommittal reference to a special artifice which could be used as a rigid upper surface and which is said to have produced the same results as the free surface. Be that as it may, all results presented by Bénard were obviously obtained with the free surface. The surface of the fluid was cooled by ambient air, which means that the cooling was not really uniform, since the ambient air convects itself when heated by the warm fluid layer. The fluid used for the entire series of steady-state experiments was spermaceti, a whale oil, which is a waxy, rigid substance at room temperature, but melts at 46°C. Molten spermaceti is a nonvolatile, viscous fluid of poor thermal conductivity; its thermal conductivity is about a thousandth of the thermal conductivity of the bottom plates. That ensures the uniformity of the temperature at the bottom.

Bénard's apparatus, which was quite simple, is shown in Fig. 1.1. The depth of the fluid was of the order of 1 mm, which is for future considerations very significant. The fluid layer was circular and about 20 cm in diameter, so that the aspect ratio, i.e. the ratio of the diameter to the depth of the fluid, was 200, thereby approximating a layer of infinite horizontal extent. The bottom plate was either brass (0.5 cm thick) or cast iron (7.5 cm thick) and was heated uniformly from below by steam from boiling water. The temperature at the bottom of the fluid was therefore 100°C. Assuming that the temperature of the ambient air was 20°C, there was then a temperature difference across the fluid layer of about 80°C, which is

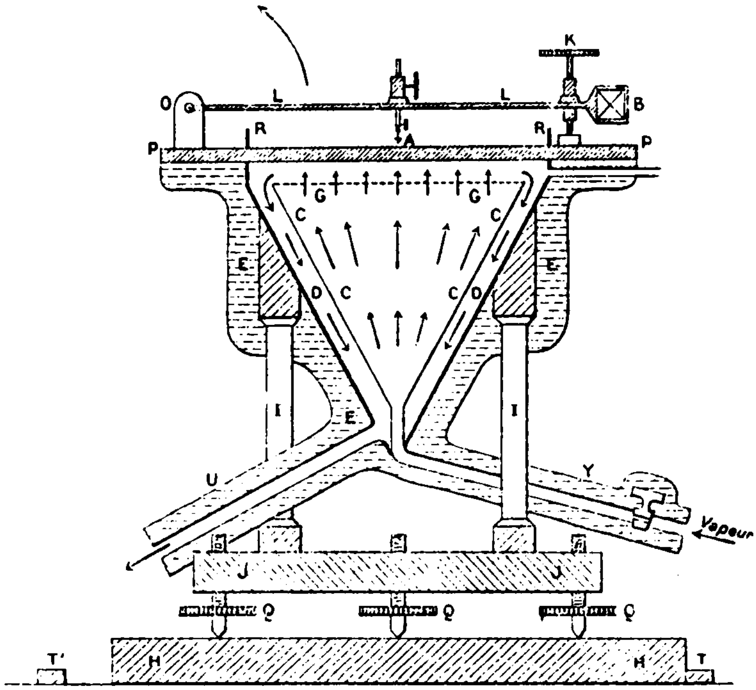

Fig. 1.1. Bénard's convection apparatus. After Bénard (1901).

much higher than the temperature difference usually applied in modern experiments. Since the depth of the fluid in Bénard's experiments was much smaller than the depth of the fluid layers in modern experiments, the temperature gradients across the fluid in Bénard's experiments were about a hundred times or more larger than the temperature gradients in most present-day convection experiments.

The initial conditions of Bénard's experiments are not spelled out. Heating of the fluid layer probably started by circulating steam underneath the bottom plate. That meant a very rapid increase of the bottom temperature to 100°C. This caused the appearance of a transient state of the fluid, which Bénard described but which we shall not consider. The transient state was followed by a steady state ("le régime permanent"), which produced the fundamental results of his experiments.

The fluid motions were made visible in three ways. Most often visualization was accomplished by the suspension of fine particles in the fluid. The particles were either aluminum or graphite powder. Occasionally Bénard

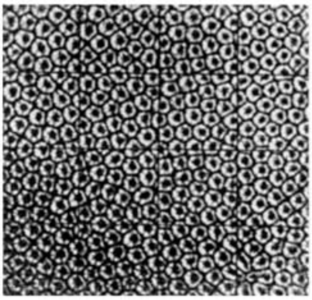

Fig. 1.2. Original photograph showing Bénard cells in a layer of spermaceti 0.810 mm deep. Visualization with graphite powder. Actual size. After Bénard (1900).

made the surface flow visible by fine floating particles, for which he used grains of lycopodium. He also applied various optical methods to make the flow visible; the optical method used most often is what we now call the shadowgraph technique.

The prime result of Bénard's experiments was the discovery of a stable, steady-state, regular pattern of hexagonal convection cells. He was obviously fascinated by the cells and believed that perfectly regular patterns existed. There are several figures in his papers showing patterns of hexagonal cells. Some of these patterns appear probably more regular than they were, because the outline of the cells was traced on photographs of shadowgraphs which seem to have been slightly out of focus. We reproduce in Fig. 1.2 a photograph which shows a less regular but perhaps more common cellular pattern. The cells in Fig. 1.2 are often polygonal; only a few are genuine hexagons. However assuming that one can realize perfect experimental conditions (which is not so easy), then one can believe in the existence of perfect patterns of hexagonal cells.

Investigation of the flow in the interior of the cells revealed that the fluid was ascending in the centers of the cells and descending along the hexagonal outline. Further optical measurements showed that the fluid surface was depressed over the centers of the cells. The depression was very small, of the order of 1 μm = 10^{-3} mm for a 1-mm-deep layer of spermaceti at 100°C. A section through a hexagonal cell with the circulation in the fluid and an exaggerated depression of the surface is shown in Fig. 1.3.

Bénard discussed possible reasons for the depression of the surface and mentioned in this context surface tension. He stated that "surface tension by itself causes already a depression above the center of the cells" (1901, p. 92). For future reference we note that Bénard also considered the variation of

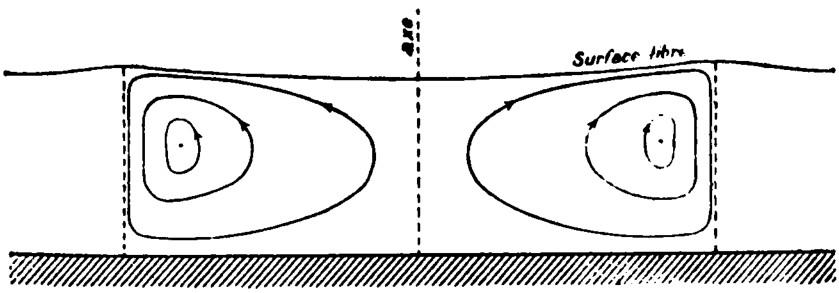

Fig. 1.3. Section through a hexagonal convection cell. The depression of the fluid surface over the center of the cell is exaggerated by a factor of 100. After Bénard (1900).

surface tension with temperature, but not in the context of the cause of the formation of the cells.

Bénard tried to determine the velocity of the flow in the interior of the hexagonal cells, as well as to find a characteristic measure of the size of the cells. The velocity measurements seem, from a modern point of view, to be useless because it was not specified at which location in the cell the measurements of the period of the circulation of tracer particles was made. The determination of the cell size, however, is still relevant because Bénard normalized the cell size with the depth of the fluid layer. As characteristic length he used the distance between the centers of two neighboring cells, which he designated by the letter λ. He searched for the laws of the variation of λ as a function of the fluid depth, the heat flux, and the temperature.

Concerning the variation of the cell dimensions as a function of the depth of the fluid, Bénard formulated a law according to which, in a first approximation, ''the cell prisms remain geometrically similar, if the depth is varied'' (1900, p. 1321). In a formula, λ/d = const. This is what, with hindsight, we expect under critical conditions. Bénard added almost immediately after the quoted law that ''precise measurements reveal a systematic deviation'' from this law. He found that in spermaceti at 100°C bottom temperature, the ratio of λ/d increased if the depth of the layer increased, namely from $\lambda/d = 3.378$ at $d = 0.440$ mm to $\lambda/d = 4.049$ at $d = 0.853$ mm. This observation is consistent with our present knowledge that the wavelength of convective motions in a fluid layer between two rigid parallel plates increases with increased supercritical conditions. Since Bénard measured the cell sizes in the different fluid layers usually at the same temperature difference (100°C at the bottom minus the temperature of the ambient air), the Rayleigh numbers of the different fluid layers differ, being large for the deep layers and smaller for the less deep layers. Our empirical knowledge from modern experiments tells us that the cell size of supercritical convective motions between two parallel plates is larger for large Rayleigh numbers (in this case for deep fluid layers) and smaller when the Rayleigh number is smaller (in this case in less deep layers), as Bénard observed.

Bénard could also vary the temperature difference across the fluid by permitting the big cast iron block on the bottom of the fluid to cool down. The effect of the variation of the temperature difference on the size of the cells is shown in various graphs, e.g. in Fig. 23 in Bénard (1900). It can be seen there that the cell size for one and the same fluid depth decreased overall as the temperature difference across the fluid decreased, in agreement with what we have since learned about the variation of the wavelength of convective motions as a function of the temperature difference. There was, however, a puzzling aspect of these cell size measurements. Bénard found that, near the point where the pattern disappeared, the cell size reached a minimum and increased a little when the temperature difference was decreased further, until the pattern disappeared. His observations of the variation of the cell size, in particular the puzzling minimum cell size, were correct but were confirmed only 90 years later.

The observations of the disappearance of the pattern at small temperature differences bear on the question of whether Bénard had detected the existence of a critical temperature difference in his experiments. He observed for four different fluid depths that the ratio λ/d was very nearly the same ($\lambda/d = 3.584 \pm 0.039$) whenever the cell pattern disappeared. Disappearance of the pattern means in modern terms that the layer had cooled down to the critical temperature difference. We know now that at the critical temperature difference the pattern has a unique wavelength $(\lambda/d)_c$, which is independent of the depth of the fluid layer. This is just what Bénard saw. But he did not realize that this was the consequence of a critical temperature difference, a concept introduced later by Rayleigh (1916a). Bénard attributed the disappearance of the pattern to the beginning solidification of the spermaceti. Much later on, Bénard (1930) was very skeptical about the concept of a critical temperature difference, although he had actually observed it when the pattern disappeared in his experiments.

All in all, Bénard's experiments provided a good basis for further experimental and theoretical work. His discovery of the hexagonal convection cells has been a lasting contribution to science. It is an irony of fate that, in spite of the quality of his work, his experiments created a monumental misconception by focusing the interest primarily on the hexagonal cells, which have, as was learned only 50 years later, little to do with buoyancy.

However at that time heating from below appeared to be the cause of the formation of the cells.

In the conclusion of his first paper Bénard referred to the interest that the hexagonal cells had found among the ''naturalists.'' This interest has continued until today. However the obvious similarity between Bénard cells and living cells has yet to prove to be a fruitful connection. In view of the complexity of living cells and the difficulties which had to be overcome in order to explain even the ''simple'' Bénard cells, it is not surprising that a real connection between both phenomena has not yet been made. Practically the same can be said about the importance of convection for the ''atmospheric circulation,'' to which Bénard refers in the introduction of his first paper.

We shall now see how the results of Bénard's experiments have been explained, and what else we have learned about convection in a fluid layer heated from below.