This book is concerned with the theory of unbounded derivations in C*-algebras, a subject whose study was motivated by questions in quantum physics and statistical mechanics, and to which the author has made a considerable contribution. This is an active area of research, and one of the most ambitious aims of the theory is to develop quantum statistical mechanics within the framework of the C*-theory. The presentation, which is based on lectures given by the author in Newcastle upon Tyne and Copenhagen, concentrates on topics from quantum statistical mechanics and differentiations on manifolds. One of the goals is to formulate the absence theorem of phase transitions in its most general form within the C* setting. For the first time, the author constructs, within that global setting, derivations for a fairly wide class of interacting models, and presents a new axiomatic treatment for the construction of time evolutions and KMS states.

The wealth of new insight offered here will make the book essential reading for graduate students and professionals working in operator algebras, mathematical physics and functional analysis.

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ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

Operator algebras in dynamical systems

The theory of unbounded derivations in C*-algebras

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CONTENTS

PREFACE

Derivations appeared for the first time at a fairly early stage in the young field of C*-algebras, and their study continues to be one of the central branches in the field. During the past four decades, the study of derivations has made great strides. Their theory divides naturally into two major parts: bounded derivations and unbounded derivations. About thirty years ago, Kaplansky (in an excellent survey [97] on derivations) brought together two apparently unrelated results which stimulated research on continuous derivations. The first, related to quantum mechanics and due to Wielandt [190] in 1949, proved that the commutation relation $ab - ba = 1$ cannot be realized by bounded operators. The second, involving differentiation and due to Šilov [180] in 1947, proved that if a Banach algebra A of continuous functions on the unit interval contains all infinitely differentiable functions, then A contains all n-times differentiable functions for some n.

It is noteworthy that Kaplansky's observations three decades ago are still applicable to recent developments in the theory of unbounded derivations, although one has to replace quantum mechanics by all of quantum physics. Furthermore, the work of Šilov continues to have a strong influence on the study of unbounded derivations in commutative C*-algebras.

At an early stage, mathematicians devoted most of their efforts to the study of bounded derivations, rather than unbounded ones, even though the work of Šilov and Wielandt had already suggested the importance of unbounded derivations. This, of course, is understandable because bounded derivations are more easily handled than unbounded ones. The study of bounded derivations has led to a beautiful mathematical theory that provides (among other things) essential tools for the study of unbounded derivations. In contrast to the study of bounded derivations, which is now approaching completion, the study of unbounded derivations is still in an early stage. Their study began much later and initially was motivated by the problem of constructing the dynamics in statistical mechanics. It soon became apparent that the work of Šilov was also important in the study of unbounded drivations in commutative C*-algebras.

The contents of the present book are based in large part on the author's lecture notes on the theory of unbounded derivations in C*-algebras given at the Universities of Copenhagen and Newcastle upon Tyne in 1977 [170]; also included are some of the extensive new developments discovered since 1977. Unbounded derivations is a fast growing field and the subjects it involves are quite diverse. Therefore I have made no attempt to give complete coverage of the theory. Rather, I have taken a somewhat personal stand on the selection of material. This selection is concentrated, for the most part, on the topics involving quantum statistical mechanics and differentiations on manifolds.

There is a good possibility that the theory of quantum lattice systems in statistical mechanics may also be developed naturally within the context of unbounded derivations in C*-algebras (although its phase transition has not yet been established even for the three-dimensional Heisenberg ferromagnet). In fact, many theorems in the theory of quantum lattice systems have already been formulated for normal *-derivations in UHF algebras. One of the most ambitious programs in the theory of unbounded derivations is to develop quantum statistical mechanics within the C*-theory. Of particular importance is the generalization of phase transition theory to quantum lattice systems. The prospect of success is not necessarily gloomy, because the absence theorem for the phase transition theory (in one-dimensional quantum lattice systems with bounded surface energy) has already been successfully formulated in the quite general setting of UHF algebras ([5], [104], [168], [169]). This 'absence theorem' refers to a result where, in a restricted context, the temperature states can be shown to be unique. Hence the 'absence' of phase transitions. One of the goals of the present book will be to formulate the absence theorem for phase transitions in its most general form within the C*-setting. Compared with the theory of quantum lattice systems, the theory of continuous quantum systems is somewhat incomplete. Except for a few examples, time evolutions have never been constructed for interesting interaction models. As a matter of fact, even derivations have never been constructed globally.

In the present book, we shall construct derivations globally for a fairly wide class of interaction models in the C*-setting. Also, an axiomatic treatment of the construction of time evolutions and KMS states will be attempted. However, it is not thoroughly discussed, and much is left to future research.

A few words concerning the theory of unbounded derivations in commutative C*-algebras: I have not attempted to give complete coverage of recent important developments. (The reason for this is that the results are so numerous that it would be more suitable to publish another book concentrated on the subject.) Instead, I have included comments on matters related to transformation groups on locally euclidean spaces because I believe it may turn out to be one of the most important problems in the subject for future research.

With respect to non-commutative manifolds, there is an excellent book by Bratteli [19], so I will not discuss them in the present work.

References to theorems refer to the section number for example Theorem 5.2.2 refers to the theorem in Section 5.2.2.

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