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Geometry of Low-dimensional Manifolds

1: Gauge Theory and Algebraic Surfaces

Proceedings of the Durham Symposium, July 1989

Edited by
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CAMBRIDGE
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011–4211, USA
10 Stamford Road, Oakleigh, Victoria 3166, Australia

First published 1990
Reprinted 1992

Printed in Great Britain at the University Press, Cambridge

Library of Congress cataloguing in publication data available

British Library cataloguing in publication data available

ISBN 0 521 39978 5 paperback

CAMBRIDGE

Cambridge University Press
978-0-521-39978-4 - Geometry of Low-dimensional Manifolds 1: Gauge Theory and Algebraic Surfaces
Edited by S. K. Donaldson and C. B. Thomas
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Cambridge University Press
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INTRODUCTION

In the past decade there have been a number of exciting new developments in an area lying roughly between *manifold theory* and *geometry*. More specifically, the principal developments concern:

(1) geometric structures on manifolds,
(2) symplectic topology and geometry,
(3) applications of Yang-Mills theory to three- and four-dimensional manifolds,
(4) new invariants of 3-manifolds and knots.

Although they have diverse origins and roots spreading out across a wide range of mathematics and physics, these different developments display many common features—some detailed and precise and some more general. Taken together, these developments have brought about a shift in the emphasis of current research on manifolds, bringing the subject much closer to geometry, in its various guises, and physics.

One unifying feature of these geometrical developments, which contrasts with some geometrical trends in earlier decades, is that in large part they treat phenomena in specific, low, dimensions. This mirrors the distinction, long recognised in topology, between the flavours of "low-dimensional" and "high-dimensional" manifold theory (although a detailed understanding of the connection between the special roles of the dimension in different contexts seems to lie some way off). This feature explains the title of the meeting held in Durham in 1989 and in turn of these volumes of Proceedings, and we hope that it captures some of the spirit of these different developments.

It may be interesting in a general introduction to recall the the emergence of some of these ideas, and some of the papers which seem to us to have been landmarks. (We postpone mathematical technicalities to the specialised introductions to the six separate sections of these volumes.) The developments can be said to have begun with the lectures [T] given in Princeton in 1978-79 by W.Thurston, in which he developed his "geometrisation" programme for 3-manifolds. Apart from the impetus given to old classification problems, Thurston's work was important for the way in which it encouraged mathematicians to look at a manifold in terms of various concomitant geometrical structures. For example, among the ideas exploited in [T] the following were to have perhaps half-suspected fall-out: representations of link groups as discrete subgroups of $PSL_2(\mathbf{C})$, surgery compatible with geometric structure, rigidity, Gromov's norm with values in the real singular homology, and most important of all, use of the theory of Riemann surfaces and Fuchsian groups to develop a feel for what might be true for special classes of manifolds in higher dimensions.

Meanwhile, another important signpost for future developments was Y. Eliashberg's proof in 1981 of "symplectic rigidity"– the fact that the group of symplectic diffeomorphisms of a symplectic manifold is C^0-closed in the full diffeomorphism group.

This is perhaps a rather technical result, but it had been isolated by Gromov in 1970 as the crux of a comprehensive "hard versus soft" alternative in "symplectic topology": Gromov showed that if this rigidity result was not true then any problem in symplectic topology (for example the classification of symplectic structures) would admit a purely algebro-topological solution (in terms of cohomology, characteristic classes, bundle theory etc.) Conversely, the rigidity result shows the need to study deeper and more specifically geometrical phenomena, beyond those of algebraic topology.

Eliashberg's original proof of symplectic rigidity was never fully published but there are now a number of proofs available, each using new phenomena in symplectic geometry as these have been uncovered. The best known of these is the "Arnol'd Conjecture" [A] on fixed points of symplectic diffeomorphisms. The original form of the conjecture, for a torus, was proved by Conley and Zehnder in 1982 [CZ] and this established rigidity, since it showed that the symplectic hypothesis forced more fixed points than required by ordinary topological considerations. Another demonstration of this rigidity, this time for contact manifolds, was provided in 1982 by Bennequin with his construction [B] of "exotic" contact structures on $\mathbf{R}^3$.

Staying with symplectic geometry, but moving on to 1984, Gromov [G] introduced "pseudo-holomorphic curves" as a new tool, thus bringing into play techniques from algebraic and differential geometry and analysis. He used these techniques to prove many rigidity results, including some extensions of the Arnol'd conjecture and the existence of exotic symplectic structures on Euclidean space. (Our "low-dimensional" theme may appear not to cover these developments in symplectic geometry, which in large part apply to symplectic manifolds of all dimensions: what one should have in mind are the crucial properties of the *two-dimensional* surfaces, or pseudo-holomorphic curves, used in Gromov's theory. Moreover his results seem to be particularly sharp in low dimensions.)

We turn now to 4-manifolds and step back two years. At the Bonner Arbeitstagung in June 1982 Michael Atiyah lectured on Donaldson's work on smooth 4-manifolds with definite intersection form, proving that the intersection form of such a manifold must be "standard". This was the first application of the "instanton" solutions of the Yang-Mills equations as a tool in 4-manifold theory, using the moduli space of solutions to provide a cobordism between such a 4-manifold and a specific union of $\mathbf{CP}^2$'s [D]. This approach again brought a substantial amount of analysis and differential geometry to bear in a new way, using analytical techniques which were developed shortly before. Seminal ideas go back to the 1980 paper [SU] of Sacks and Uhlenbeck. They showed what could be done with non-linear elliptic problems for which, because of conformal invariance, the relevant estimates lie on the borderline of the Sobolev inequalities. These analytical techniques are relevant both in the Yang-Mills theory and also to pseudo-holomorphic curves. Other important and influential analytical techniques, motivated in part by Physics, were developed by C.Taubes [Ta].

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Combined with the topological h-cobordism theorem of M. Freedman, proved shortly before, the result on smooth 4-manifolds with definite forms was quickly used to deduce, among other things, that $\mathbf{R}^4$ admits exotic smooth structures. Many different applications of these instantons, leading to strong differential-topological conclusions, were made in the following years by a number of mathematicians; the other main strand in the work being the definition of new invariants for smooth 4-manifolds, and their use to detect distinct differentiable structures on complex algebraic surfaces (thus refuting the smooth h-cobordism theorem in four dimensions).

From an apparently totally different direction the *Jones polynomial* emerged in a series of seminars held at the University of Geneva in the summer of 1984. This was a new invariant of knots and links which, in its original form [J], is defined by the traces of a series of representations of the Braid Groups which had been encountered in the theory of von Neumann algebras, and were previously known in statistical mechanics. For some time, in spite of its obvious power as an invariant of knots and links in ordinary space, the geometric meaning of the Jones invariant remained rather mysterious, although a multitude of connections were discovered with (among other things) combinatorics, exactly soluble models in statistical physics and conformal field theories.

In the spring of the next year, 1985, A. Casson gave a series of lectures in Berkeley on a new integer invariant for homology 3-spheres which he had discovered. This Casson invariant "counts" the number of representations of the fundamental group in $SU(2)$ and has a number of very interesting properties. On the one hand it gives an integer lifting of the well-established Rohlin $\mathbf{Z}/2$ μ-invariant. On the other hand Casson's definition was very geometric, employing the moduli spaces of unitary representations of the fundamental groups of surfaces in an essential way. (These moduli spaces had been extensively studied by algebraic geometers, and from the point of view of Yang-Mills theory in the influential 1982 paper of Atiyah and Bott [AB].) Since such representations correspond to flat connections it was clear that Casson's theory would very likely make contact with the more analytical work on Yang-Mills fields. On the other hand Casson showed, in his study of the behaviour of the invariant under surgery, that there was a rich connection with knot theory and more familiar techniques in geometric topology. For a very readable account of Cassons work see the survey by A. Marin [M].

Around 1986 A. Floer introduced important new ideas which applied both to symplectic geometry and to Yang-Mills theory, providing a prime example of the interaction between these two fields. Floer's theory brought together a number of powerful ingredients; one of the most distinctive was his novel use of ideas from Morse theory. An important motivation for Floer's approach was the 1982 paper by E. Witten [W1] which, among other things, gave a new analytical proof of the Morse inequalities and explained their connection with instantons, as used in Quantum Theory.

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In symplectic geometry one of Floer's main acheivements was the proof of a generalised form of the Arnol'd conjecture [F1]. On the Yang-Mills side, Floer defined new invariants of homology 3-spheres, the *instanton homology groups* [F2]. By work of Taubes the Casson invariant equals one half of the Euler characteristic of these homology groups. Their definition uses moduli spaces of instantons over a 4-dimensional tube, asymptotic to flat connections at the ends, and these are interpreted in the Morse theory picture as the gradient flow lines connecting critical points of the Chern-Simons functional.

Even more recently (1988), Witten has provided a quantum field theoretic interpretation of the various Yang-Mills invariants of 4-manifolds and, in the other direction, has used ideas from quantum field theory to give a purely 3-dimensional definition of the Jones link invariants [W2]. Witten's idea is to use a functional integral involving the Chern-Simons invariant and holonomy around loops, over the space of all connections over a 3-manifold. The beauty of this approach is illustrated by the fact that the choices (quantisations) involved in the construction of the representations used by Jones reflect the need to make this integral actually defined. In addition Witten was able to find new invariants for 3-manifolds.

It should be clear, even from this bald historical summary, how fruitful the cros-fertilisation between the various theories has been. When the idea of a Durham conference on this area was first mooted, in the summer of 1984, the organisers certainly intended that it should cover Yang-Mills theory, symplectic geometry and related developments in theoretical physics. However the proposal was left vague enough to allow for unpredictable progress, sudden shifts of interest, new insights, and the travel plans of those invited. We believe that the richness of the contributions in both volumes has justified our approach, but as always the final judgement rests with the reader.

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Acknowledgements

We should like to take this opportunity to thank the London Mathematical Society and the Science and Engineering Research Council for their generous support of the Symposium in Durham. We thank the members of the Durham Mathematics Department, particularly Professor Philip Higgins, Dr. John Bolton and Dr. Richard Ward, for their work and hospitality in putting on the meeting, and Mrs. S. Nesbitt and Mrs. J. Gibson who provided most efficient organisation. We also thank all those at Grey College who arranged the accommodation for the participants. Finally we should like to thank Dieter Kotschick and Lisa Jeffrey for writing up notes on some of the lectures, which have made an important addition to these volumes.