

Cambridge University Press

0521397804 - The Thermomechanics of Plasticity and Fracture - Gerard A. Maugin
Frontmatter

[More information](#)

The thermomechanics of plasticity and fracture

Cambridge University Press

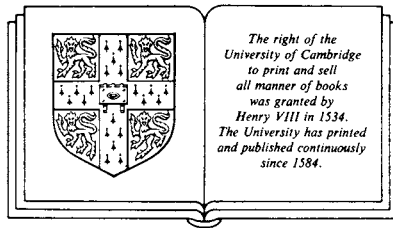
0521397804 - The Thermomechanics of Plasticity and Fracture - Gerard A. Maugin

Frontmatter

[More information](#)

The thermomechanics of plasticity and fracture

GERARD A. MAUGIN

*Modélisation en Mécanique**Université Pierre et Marie Curie (Paris VI)*

CAMBRIDGE UNIVERSITY PRESS

Cambridge

New York Port Chester

Melbourne Sydney

Cambridge University Press

0521397804 - The Thermomechanics of Plasticity and Fracture - Gerard A. Maugin

Frontmatter

[More information](#)

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, Victoria 3166, Australia

© Cambridge University Press 1992

First published 1992

British Library cataloguing in publication data

Maugin, Gérard A. 1944–

The thermomechanics of plasticity and fracture.

1. Plasticity, Mathematics

I. Title

531.38

Library of Congress cataloguing in publication data available

ISBN 0 521 39476 7 hardback

ISBN 0 521 39780 4 paperback

Transferred to digital printing 2003

Contents

Preface	<i>page xi</i>
Historical perspective	xv
Notation	xvii
1 Introduction to plasticity: experimental facts	1
1.1 Elastic and plastic behaviours	1
1.2 Influence of the strain rate	7
1.3 Other effects	9
1.4 The plastic-hardening threshold: experimental data	11
1.5 Examples of plastic-flow criteria	17
1.6 Conclusions: working hypotheses in elastoplasticity	24
Problems for Chapter 1	25
2 Thermomechanics of elastoviscoplastic continua	30
2.1 The small-perturbation hypothesis	30
2.2 General principles of continuous-media thermomechanics	32
2.2.1 Principle of virtual power (PVP)	32
2.2.2 Principles (laws) of thermodynamics for continuous media	36
2.3 Using the Clausius–Duhem inequality	38
2.4 Particular cases of solid media	40
2.4.1 Neither plastic strain nor associated phenomena	40
2.4.2 Maxwell’s viscoelasticity	41
2.4.3 Thermoelasticity	43
2.4.4 The difference between viscous and plastic phenomena	43
Problems for Chapter 2	46
3 Small-strain elastoplasticity	50
3.1 Reminder of the thermomechanical formulation	50
3.1.1 The notion of normal dissipative mechanism	50
3.1.2 Dissipation pseudo-potential	52
3.1.3 Positively homogeneous dissipation functions of degree 1	53
3.2 Perfect plasticity equations in SPH	54

vi	Contents	
3.3	Incremental nature of the elastoplasticity laws	57
3.4	Remarks	61
3.4.1	Energy aspect	61
3.4.2	Thermodynamic restriction on the convex C	62
3.4.3	Regularity	62
3.4.4	The Prandtl–Reuss relations	62
3.4.5	The Lévy–Mises relations	63
3.4.6	The Hencky–Nadai relations	63
3.5	Viscoplasticity	63
	Problems for Chapter 3	65
4	Problems in perfect elastoplasticity	69
4.1	Reminder of the perfect elastoplasticity equations	69
4.2	Problem in terms of velocities	72
4.2.1	The intuitive viewpoint	72
4.2.2	The Greenberg minimum principle	72
4.2.3	The Hodge–Prager minimum principle	73
4.2.4	Mathematical analysis of quasi-static evolution	77
4.2.5	Evolution in stresses	77
4.2.6	Evolution of plastic strains	80
4.3	Asymptotic behaviour: shakedown	81
4.3.1	Practical motivation	81
4.3.2	The Melan–Koiter Theorem	82
4.4	Remark on discontinuities	85
	Appendix to Chapter 4: minimum principles in elasticity	85
	Problems for Chapter 4	89
5	Elastoplasticity with strain-hardening	94
5.1	Generalized standard media	94
5.1.1	The basic idea	94
5.1.2	Generalization	96
5.1.3	Examples	97
5.2	Relations between velocities; incremental constitutive equations	107
5.3	Stability in Ilyushin’s sense	111
5.4	Elastoplastic evolution in the presence of hardening	113
5.5	Simplified abstract formulation	114
	Problems for Chapter 5	116
6	Elements of limit analysis	121
6.1	The notion of limit load	121
6.1.1	Characterization of the limit load	122
6.1.2	Case of the rigid–plastic model	123
6.1.3	Example: spherical envelope under pressure	123

Contents	vii
6.2 Computation of the limit load	127
6.2.1 Generalities	127
6.2.2 Static method	129
6.2.3 Kinematic method	130
6.3 Example of a foundation's limit load	130
Problems for Chapter 6	132
7 Crack propagation and fracture mechanics	136
7.1 Introduction and elementary notions	136
7.2 The notion of singularity	139
7.3 The energy aspect of brittle fracture	144
7.4 The Rice–Eshelby–Cherepanov integral	148
7.5 Global potential, generalized Rice integral, energy-release rate	149
7.6 Quasi-static evolution of a crack system in an elastic solid in brittle fracture	150
7.7 Similarity between plasticity and fracture	153
7.8 The Barenblatt theory	155
7.9 Introduction of a plastic zone	156
Problems for Chapter 7	158
8 Elastoplasticity with finite strains	162
8.1 Decomposition of elastoplastic strains	162
8.2 Green–Naghdi decomposition	166
8.3 Lee decomposition	167
8.4 Evolution equation (normality rule)	169
Problems for Chapter 8	171
9 Homogenization of elastoplastic composites	174
9.1 Notion of homogenization	174
9.2 Notion of representative volume element and localization	175
9.2.1 Representative volume element (RVE)	175
9.2.2 Localization process	176
9.2.3 The Hill–Mandel principle of macro-homogeneity	177
9.2.4 Functional notation	178
9.3 The example of pure elasticity	179
9.3.1 The localization problem	179
9.3.2 Case where \mathcal{E} is prescribed	180
9.3.3 Case where Σ is prescribed	181
9.3.4 Equivalence between ‘prescribed stress’ and ‘prescribed strain’	182

viii	Contents	
9.4	Elastoplastic constituents	183
	9.4.1 Macroscopic potentials	183
	9.4.2 Stability in the sense of Drucker	185
	9.4.3 Macroscopic loading surface, macroscopic 'convex'	186
9.5	Structure of macroscopic constitutive equations	187
	9.5.1 State variables	187
	9.5.2 Internal energy of the macroscopic material	188
	9.5.3 Equations of state	188
	9.5.4 Example of an approximate model	189
9.6	First example: composite with unidirectional fibres	191
9.7	Second example: polycrystals	194
	9.7.1 The monocrystal	194
	9.7.2 The polycrystal	195
9.8	Notion of limit analysis for composites	197
	9.8.1 Extremal flow surface	197
	9.8.2 Determination of homogenized plastically admissible stresses	198
9.9	Homogenization of cracked materials	199
	Problems for Chapter 9	202
10	Coupling between plasticity and damage	206
10.1	Notion of damage	206
10.2	Thermodynamic formulation in SPH	207
10.3	Elastoplasticity of a damaged body	208
	10.3.1 Damage criterion	208
	10.3.2 Evolution of damage parameters	210
	10.3.3 Plastic microstrains	211
	10.3.4 Coupling with plasticity	212
10.4	Example of a complete model with ductile damage	213
	Problems for Chapter 10	214
11	Numerical solution of plasticity problems	219
11.1	Introduction	219
11.2	Elementary notions on numerical computations	220
11.3	Application to elastoplasticity	220
	11.3.1 Explicit scheme	220
	11.3.2 Implicit scheme	221
	11.3.3 Incremental problem for the implicit scheme	223
	11.3.4 Example of iterative method (Ilyushin)	223
	11.3.5 An elastoplastic thin flat plate with a thermal loading	226
11.4	Application of the finite-element method	228
11.5	Examples of computations by FEM in elastoplasticity	230

Contents	ix
11.5.1 Elastoplastic torsion of a cylindrical rod with a multiconnected section	230
11.5.2 Traction of a cracked rectangular plate	233
Problems for Chapter 11	235
12 Experimental study using infrared thermography	245
12.1 Heat equation in a deformable solid	245
12.2 Linearization about a natural reference state	248
12.3 Method of infrared thermography	250
12.4 Temperature distribution in fracture	252
12.4.1 Consequences of thermodynamic laws	252
12.4.2 Singularity of the temperature distribution	256
12.5 Illustrative examples	258
Problems for Chapter 12	258
Appendix 1 Thermodynamics of continuous media	262
A1.1 General notions	262
A1.1.1 Thermodynamic systems	262
A1.1.2 Thermodynamic state variables	263
A1.1.3 Thermodynamic state	263
A1.2 Thermostatistics	264
A1.2.1 Axioms of thermostatistics	264
A1.2.2 Scaling of temperature, Carnot's Theorem	267
A1.2.3 Thermodynamic potentials	268
A1.2.4 The evolution of real systems	270
A1.3 Thermodynamics	272
A1.3.1 The theory of irreversible processes	272
A1.3.2 The 'rational' theory of Coleman and Noll	276
A1.3.3 Theory with internal variables	276
A1.3.3.1 General properties	276
A1.3.3.2 Local accompanying state	278
A1.3.3.3 Evolution laws for internal variables	280
Appendix 2 Convexity	283
A2.1 Definitions	283
A2.1.1 Convex function	283
A2.1.2 Indicator function of a convex set	283
A2.2 Subdifferentials	285
A2.3 Lower semicontinuity	288
A2.4 Conjugate functions, Legendre–Fenchel transformation	288
A2.5 Minimization of functions	291

Appendix 3 Analytic solutions of some problems in elastoplasticity	293
A3.1 Elastoplastic loading of a wedge	293
A3.1.1 General equations	293
A3.1.2 Elastic law of state	294
A3.1.3 Prandtl–Reuss equations	295
A3.1.4 The full elastic solution	295
A3.1.5 Elastoplastic border	296
A3.1.6 The elastoplastic solution	297
A3.2 Elastoplastic torsion of a circular shaft	299
A3.2.1 Elastic solution	299
A3.2.2 Elastoplastic solution	300
A3.3 Tube subjected to combined torsion and simple traction	302
A3.4 Cyclic torsion of a composite with unidirectional fibres	304
A3.4.1 Basic equations	305
A3.4.2 Torsion of a slice	306
A3.4.3 Material obeying a generalized Tresca criterion	308
A3.5 Problems with hardening	309
Appendix 4 Analytic computation of stress-intensity factors	313
A4.1 Plane problems in isotropic linear elasticity	313
A4.2 Stress-intensity factor at the crack tip	316
A4.3 Remark on numerical computations of stress-intensity factors	321
Further reading	324
Bibliography	326
Index	341

Preface

The present book is an outgrowth of my lecture notes for a graduate course on 'Plasticity and fracture' delivered for the past five years to students in Theoretical Mechanics and Applied Mathematics at the Pierre-et-Marie Curie University in Paris. It also corresponds to notes prepared for an intensive course in modern plasticity to be included in a European graduate curriculum in Mechanics. It bears the imprint of a theoretician, but it should be of equal interest to practitioners willing to make an effort on the mathematical side. The prerequisites are standard and include classical (undergraduate) courses in applied analysis and Cartesian tensors, a basic course in continuum mechanics (elasticity and fluid mechanics), and some knowledge of the strength of materials (for exercises with a practical touch), of numerical methods, and of elementary thermodynamics. More sophisticated thermodynamics and elements of convex analysis, needed for a good understanding of the contents of the book, are recalled in Appendices.

The book deals specifically with what has become known as the *mathematical* theory of plasticity and fracture as (unduly) opposed to the *physical* theory of these fields. The first expression is reserved for qualifying the macroscopic, phenomenological approach which proposes equations abstracted from generally accepted experimental facts, studies the adequacy of the consequences drawn from these equations to those facts, cares for the mathematical soundness of these equations (do they have nice properties?), and then, with some confidence, provides useful tools to designers and engineers. The second expression refers to the 'physical' approach which consists in justifying laws governing macroscopic phenomena on the basis of microscopic descriptions at a finer scale (that of the discrete world), and thus is intimately related to solid-state physics, metallurgy and, more generally, what is now called materials science. For example, with such a frame of mind, the slip deformation in crystal plasticity is viewed as resulting from the movement and generation of dislocations, and is generally irreversible due to the atomic potential barriers. Although my own research would incline me to consider this stimulating aspect of plastic phenom-

ena, the present book deals practically exclusively with the macroscopic, smoothed-out scale, physics being used in the guise of irreversible thermodynamics in order to control the macroscopic evolution along the 'Arrow of time'. For the physical theory of plasticity and fracture I recommend Asaro (1983), Cottrell (1953), Honeycombe (1968), Mura (1982), Seeger (1958), Teodosiu (1975, 1982), Zener (1948), and the book of Polukhin *et al.* (1983).

Numerous problems are proposed by way of illustration or to provoke the thoughts of the reader. But the composition of the book is quite different from that of some 'classics' of plasticity theory. Just to quote two examples, while Hill's book of 1950 devotes some seventy pages among its first chapters to a general presentation and general theorems (not many at that time) and then proceeds for several hundred pages to solving problems analytically, including by the slip-line theory (in rigid-plastic bodies), Kachanov's book of 1974, another classic, presents many applications and then concludes with general theorems. Since then the emphasis has shifted, this being faithfully reflected in the theoretical and numerical developments reported in the present book: with the implementation of powerful numerical methods (finite elements, fast integration of systems of evolution equations), analytical solutions have often become outdated while general theorems have gained in importance, perforce. Therefore, most of the present book is rather general in approach with theoretical statements spread all over while some analytical solutions are given because of their everlasting pedagogical value. More on this evolution can be found in Drucker (1988).

Another development reflected in the title is the *thermomechanical* framework which, to the posthumous satisfaction of the great P. Duhem, has been fully integrated into the curriculum in continuum mechanics. This also fully agrees with the late H. Ziegler's point of view (Ziegler, 1963; Ziegler and Wehrli, 1987).

Some inevitable choices had to be made as, originally, the course was delivered in ten sessions of two hours. As a consequence, I think it fair to announce at the outset what the reader will *not* find in the book: non-associated flow rules, non-convex yield surfaces, slip-line theory, stability and wave problems in elastoplasticity, and applications to soil mechanics (except for a very few exceptions). In contrast, the reader *will* find hereinafter elastoplasticity with hardening, an introduction to elastoplasticity in finite strains, homogenization of elastic-plastic composites, coupling between plasticity and damage, path-independent integrals in fracture, numerical treatments of elastoplasticity problems, coupling with thermal fields and singularities, and applications to metals and crystals. That is, all

through these pages the plastic potential coincides with the yield surface, which is always supposed to be convex: the normality rule applies and Drucker's 'postulate' holds true (see counterexamples in Houlsby (1981) and Richmond and Spitzig (1980); Mroz (1963), and Ziegler and Wehrli (1987)). Strain-rate dependence is not considered in general, but for comparison purposes. Damage and fracture enter the *same* thermodynamical framework as elastoplasticity. This owes much to the works of Nguyen Quoc Son (in elastoplasticity and fracture) and Pierre Suquet (for composites) from whom we have borrowed much material (especially, Nguyen Quoc Son (1981a) and Suquet (1987)). My admiration for those who have contributed much to this *nonlinear* science is expressed in the list of names given in the following 'Historical perspective', as well as in the Bibliography, obviously far from exhaustive but, in my opinion, doing justice in a balanced manner to North American, West European and East European contributions.

The reader may naturally wander through the pages and chapters (a practice I very much enjoy myself), but if any advice can be given at all the following order of reading may be an astute, if not efficient, one: Chapter 1, then Appendix 1, followed by Chapter 2, completed by Appendix 2, then Chapters 3 and 4 and Appendix 3, and the rest in order of appearance. Chapters 8, 9 and 10 can be skipped in a first reading. In my own practice the contents of the appendices, together with many of the problems, were left to the students as personal reading and homework or term papers.

It is hoped that the book will find its place both on the student's shelf and in the professional scientist's office. I would be most pleased if designers and engineers would consider it as a reference.

My thanks belong to those who have helped me to capture the flavour and peculiarities of elastoplasticity, for me a rather young science. I found efficiency, kindness, encouragement and advice in the CUP team on (Applied) Mathematics and Mechanics. Moreover, the 'home typist', Eleni, the only person who 'took' this course on elastoplasticity without taking first 'Continuum mechanics', has done an excellent job in typing the manuscript. Can I ask for more than that? I feel spoiled.

G.A. Maugin

Historical perspective[†]

A Landmarks in plasticity theory from 1870 to 1980:

- Tresca (1872), Barré de Saint Venant (1871): Flow criterion; torsion problem
- Bauschinger (1886): The ‘Bauschinger’ effect
- Mohr (1900): Mohr’s circle to express criterion of failure (cited by Timoshenko (1953, p. 286))
- Prandtl (1903): Membrane analogy, Prandtl–Reuss relations (Prandtl, 1924; Reuss, 1939)
- Huber (1904), Mises R. von (1913): Flow criterion
- Hencky (1923): Slip-line theory
- Hencky (1924): Hencky’s laws
- Odqvist (1933): Hardening parameter (Odqvist’s parameter)
- Melan (1938): Plastic potential
- Melan (1938): Shakedown theory (static approach)
- Bridgman (1940–1952): High-pressure deformation and flow
- Ilyushin (1943): Method of successive elastic solutions
- Zener (1948): Rheological models
- Hill (1948): Maximal dissipation principle
- Hodge, Prager (1949): Minimum principle
- Ilyushin (1948): Stability postulate
- Greenberg (1949): Minimum principle
- Drucker (1951): Drucker’s inequality
- Symonds (1951): Limit analysis, collapse mechanisms
- Koiter (1960): Shakedown theory (kinematic approach)
- Ziegler (1963): Plasticity and nonequilibrium thermodynamics, nonlinear generalization of Onsager’s relations
- Lee (1969): Multiplicative decomposition of finite deformation gradient
- Halphen and Nguyen Quoc Son (1975): Notion of generalized standard material

[†] Full references in Bibliography

Strang, Suquet, Temam (1979, 1980): Functions of bounded variations in elastoplasticity

B Landmarks in fracture theory from 1920 to 1970

Griffith (1920): Surface energy theory

Westergaard (1939): Singularity at crack tip

Irwin and Kries (1951): Elasticity theory of fracture

Eshelby (1951): Path-independent integral (also Cherepanov, 1967, Rice, 1968)

Barenblatt (1960): Elastic theory of cohesive forces at cracks

C Landmarks in damage theory (1858—1963)

Woehler (1858–1870): Cyclic loading, Woehler's curves

Norton (1929): Creep, Norton's law

Kachanov (1958): Notion of damage

Rabotnov (1963): Damage parameter

D Landmarks in related areas (useful for the subject matter of the present book)

Rayleigh, Lord (1873): Dissipation function

Goursat (1898): Biharmonic and analytic functions

Carathéodory (1909, 1925): Axiomatics of thermostatics

Kolossov (1909), Muskhelishvili (1953): Complex-function technique in plane problems

Duhem (1911): The Clausius–Duhem inequality

Sobolev (1936): function spaces of the 'Sobolev' type

Onsager, Casimir, Meixner, de Donder, etc. (1940, 1960): Nonequilibrium thermodynamics (see de Groot and Mazur (1962))

Bridgman (1943): The notion of internal variable

Fenchel (1949): Conjugacy and Legendre–Fenchel transformation (see Rockafellar (1970))

Coleman, Noll, Truesdell (1960–1970): 'Rational thermodynamics' (see Truesdell (1969))

Argyris and others (1965): Finite-element method

Notation

Vectors and Cartesian tensors

- Real line: \mathbb{R}
 Three-dimensional Euclidean space: \mathbb{E}^3
 Open set of \mathbb{R}^3 : Ω
 Boundary of Ω : $\partial\Omega$
 Closure of Ω : $\bar{\Omega}$
 Orthonormal basis in \mathbb{E}^3 : $\mathbf{e}_i \cdot \mathbf{e}_j = \delta_{ij}$; $\{\mathbf{e}_i\} i = 1, 2, 3$
 Kronecker delta (= 1 if $i = j$, = 0 otherwise): δ_{ij}
 Vector: $\mathbf{V} = \sum_{i=1}^3 V_i \mathbf{e}_i$
 Einstein summation convention (on dummy indices): $A_j B_j = \sum_{j=1}^3 A_j B_j = A_1 B_1 + A_2 B_2 + A_3 B_3$
 Second-order tensor $\boldsymbol{\sigma} = \sum_{i,j=1}^3 \sigma_{ij} \mathbf{e}_i \otimes \mathbf{e}_j = \sigma_{ij} \mathbf{e}_i \otimes \mathbf{e}_j$
 Tensor product \otimes : $(\mathbf{A} \otimes \mathbf{B})_{ij} = A_i B_j$
 Transposition $\boldsymbol{\sigma}^T = \{\sigma_{ji}\}$ if $\boldsymbol{\sigma} = \{\sigma_{ij}\}$
 Symmetric second-order tensor: $\boldsymbol{\sigma} = \boldsymbol{\sigma}^T$ or $\sigma_{ij} = \sigma_{ji}$ (six independent components at most)
 Symmetric part: $\boldsymbol{\sigma}_S \equiv \{\sigma_{(ij)} \equiv \frac{1}{2}(\sigma_{ij} + \sigma_{ji})\}$
 Anti-(or skew-) symmetric part: $\boldsymbol{\sigma}_A \equiv \{\sigma_{[ij]} = \frac{1}{2}(\sigma_{ij} - \sigma_{ji})\}$
 Scalar product of two vectors: $\mathbf{A} \cdot \mathbf{B} = \sum_{j=1}^3 A_j B_j = A_j B_j$
 Vector product of two vectors: $(\mathbf{A} \times \mathbf{B})_i = \sum_{j,k=1}^3 \varepsilon_{ijk} A_j B_k = \varepsilon_{ijk} A_j B_k$
 Permutation symbol ε_{ijk} :
$$\begin{cases} = 1 & \text{if } i, j, k \text{ is an even permutation of } 1, 2, 3 \\ = -1 & \text{if } i, j, k \text{ is an odd permutation of } 1, 2, 3 \\ = 0 & \text{otherwise} \end{cases}$$

 Trace of a tensor $\boldsymbol{\sigma}$: $\text{tr } \boldsymbol{\sigma} = \sum_{i=1}^3 \sigma_{ii} = \sigma_{11} + \sigma_{22} + \sigma_{33}$
 Deviator of a tensor: $\boldsymbol{\sigma}^d = \boldsymbol{\sigma} - \frac{1}{3}(\text{tr } \boldsymbol{\sigma}) \mathbf{1} = \{\sigma_{ij} - \frac{1}{3}\sigma_{kk} \delta_{ij}\}$
 'Inner' product of two tensors: $\boldsymbol{\sigma} : \boldsymbol{\varepsilon} = \sum_{i,j=1}^3 \sigma_{ij} \varepsilon_{ji} = \sigma_{ij} \varepsilon_{ji}$.

Differential operators:

- Nabla ('del' operator) $\nabla := \{\partial/\partial x_i; i = 1, 2, 3\}$, e.g., $\nabla\phi = \phi_{,i} \mathbf{e}_i$.
 Gradient tensor of a vector field: $\nabla \mathbf{A} = \{A_{i,j}\}$

Divergence of a vector $\nabla \cdot \mathbf{A} = \text{tr}(\nabla \mathbf{A}) = A_{i,i} = \sum_{i=1}^3 A_{i,i}$

Curl of a vector: $(\text{curl } \mathbf{A})_i = (\nabla \times \mathbf{A})_i = \varepsilon_{ijk} A_{k,j}$

Normal derivative: $\partial \phi / \partial n = (\mathbf{n} \cdot \nabla) \phi = \phi_{,i} n_i$

Laplacian of a scalar: $\nabla \cdot \nabla \phi = \nabla^2 \phi = \phi_{,ii}$

Kinematics, strains:

Position (placement) in \mathbb{E}^3 : \mathbf{x}, \mathbf{X}

Displacement vector: $\mathbf{u} = \{u_i\}$

Linear strain tensor: $\boldsymbol{\varepsilon} = (\nabla \mathbf{u})_S = \{\varepsilon_{ij} = \frac{1}{3}(u_{i,j} + u_{j,i})\}$

Deviator of $\boldsymbol{\varepsilon}$: $\boldsymbol{\varepsilon}^d$

Elastic strain: $\boldsymbol{\varepsilon}^e$

Plastic strain: $\boldsymbol{\varepsilon}^p$

Viscous strain: $\boldsymbol{\varepsilon}^v$

Equivalent strain: $\varepsilon_p = (\frac{2}{3} \varepsilon_{ij}^p \varepsilon_{ij}^p)^{1/2}$

Cumulated plastic strain: $\bar{\varepsilon}^p = \int_0^t (\frac{2}{3} \dot{\varepsilon}_{ij}^p \dot{\varepsilon}_{ij}^p)^{1/2} dt$

Velocity vector: $\mathbf{v} = \{v_i = \dot{u}_i\}$

Partial time derivative: $\dot{A} = \partial A / \partial t$

Acceleration vector: $\boldsymbol{\gamma} = \dot{\mathbf{v}} = \ddot{\mathbf{u}}$

Strain-rate tensor: $\mathbf{D} = \{D_{ij}\} = (\nabla \mathbf{v})_S$

Rotation-rate tensor: $\boldsymbol{\Omega} = \{\Omega_{ij}\} = (\nabla \mathbf{v})_A$

Vorticity vector: $\boldsymbol{\omega} = \frac{1}{2} \nabla \times \mathbf{v}$

Finite deformation: \mathbf{F}

n -vector of internal variables: α

Forces, stresses:

Stress tensor: $\boldsymbol{\sigma}$

Deviator of stress: $\mathbf{s} = \boldsymbol{\sigma}^d$

Invariants of stresses: $(\sigma_I, \sigma_{II}, \sigma_{III})$ or $I_\alpha(\boldsymbol{\sigma}), \alpha = 1, 2, 3$

Mean stress: $\sigma_m = \frac{1}{3} \text{tr } \boldsymbol{\sigma}$

Normal stress: σ_n

Tangential stress: τ

Pressure: p

Elastic limit: σ_0 (yield stress)

Mises equivalent stress: $\sigma_Y = (\frac{3}{2} \sigma_{ij}^d \sigma_{ij}^d)^{1/2}$

Convex of plasticity: C

Plastic multiplier: $\dot{\lambda}$

Viscosity: η_v

Loading function: f or F

Body force: \mathbf{f} or \mathbf{g}

Surface traction: \mathbf{T}
 Hardening parameters: α, β
 Hardening modulus: h
 Force associated to α : \mathbf{A}
 Tensor of elasticity coefficients (rigidities): \mathbf{E}
 Tensor of elastic compliances: \mathbf{S}
 Lamé coefficients: λ and μ
 Hooke's modulus: E
 Poisson's ratio: ν
 Damage parameter: D

Thermodynamics:

Thermodynamic temperature: θ
 Matter density: ρ
 Internal energy per unit mass: e
 Entropy per unit mass: η
 Free energy per unit mass: ψ
 Total internal energy: E
 Total kinetic energy: K
 Total entropy: \mathcal{N}
 Total power: \mathcal{P}
 Total dissipation: Φ
 Internal energy per unit volume: \mathcal{E}
 Entropy per unit volume: S
 Free energy per unit volume: W
 Source heat per unit mass: h
 Heat flux vector = $\{q_i\}$: \mathbf{q}
 Dissipation per unit volume: ϕ
 Viscous dissipation: ϕ_v
 Plastic dissipation: ϕ_p
 Thermal dissipation: ϕ_q
 Intrinsic dissipation: ϕ_{intr}
 Dissipation function/potential: \mathcal{D}