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Representations of Solvable Groups

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CAMBRIDGE
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First published 1993

Library of Congress cataloguing in publication data available

British Library cataloguing in publication data available

ISBN 0 521 39739 1

Transferred to digital printing 2004

CONTENTS

PREFACE

Representation theory has very strong interplay with group structure. This is particularly true for finite solvable groups G, because their chief factors are irreducible modules for G over fields of prime order. In this monograph, we present some topics and problems arising in the representation theory of solvable groups. In particular, we study modules over finite fields, yet give applications to ordinary and Brauer characters of solvable groups.

It is not our intent to develop representation theory from scratch, but rather to discuss techniques and problems in current research. On the other hand, we wish that the manuscript be accessible to a reasonably wide group of people, including advanced graduate students, working in group theory. We refer to two basic references, namely:

[Hu] B. Huppert, "Endliche Gruppen I" and
[Is] I. M. Isaacs, "Character Theory of Finite Groups".

We believe that readers fairly familiar with these texts should have little problem reading the manuscript. We do also quote some material from the first chapter appearing in the sequel to [Hu], namely Chapter VII of "Finite Groups II" by B. Huppert and N. Blackburn [HB]. That chapter is entitled "Elements of General Representation Theory". Many of the results from these sources for which we have frequent use are presented (generally without proof) in Chapter 0, "Preliminaries". Since we present some applications to block theory, we state and/or prove several related results in Chapter 0. To this end, we have quoted some material here from "Representations of Finite Groups", by H. Nagao and Y. Tsushima [NT], although many of the quoted results also appear in the sketchy introduction to block theory that

appears in the last chapter of [Is]. In our preliminary chapter, we do include proofs of Fong reduction and the Fong–Swan Theorem.

Of course, module (and character) induction is a powerful tool in representation theory, particularly when paired with Clifford's Theorem. Consequently, we need to study "quasi-primitive" linear groups, where those techniques do not apply. For solvable groups, the condition of quasi-primitivity imposes strong restrictions on the normal structure of the group. We study this extensively in Section 1, without restriction on the underlying field. An important class of solvable (quasi-primitive) linear groups over finite fields are the "semi-linear" groups. We study these in Section 2 along with conditions that force a linear group to indeed be a semi-linear group. Section 3 gives bounds for orders and derived lengths of solvable linear groups and permutation groups.

Much of Chapters II and III (Sections 4 through 11) deals with orbits of solvable linear groups or, as in Section 5, orbits of permutation groups. Of course, for solvable groups, orbit sizes of linear groups and those of permutation groups are closely related. This becomes clear in Section 6, where we give a new proof of Huppert's classification of doubly transitive solvable permutation groups. Many of the questions about orbit sizes of linear groups are related to the existence (or non-existence) of "regular" orbits. Our emphasis here again is on finite fields, because otherwise regular orbits always exist. The main feature of Chapter III, which is critical for Chapters IV and V, is the study of linear groups with "Sylow centralizers".

Chapters IV and V deal with ordinary and modular characters and their degrees. In Section 12, we prove Brauer's height-zero conjecture for solvable G, using material from Sections 5, 6, 9 and 10. In Section 15, we give a character-counting argument and use it to prove the Alperin–McKay conjecture for p-solvable groups. In Section 16, we discuss the derived length and the number of character degrees of a solvable group. This partially relies on a theorem of Berger, presented in Section 8, which unlike other orbit theorems gives the existence of small orbits.

The final chapter introduces the theory of "π-special" characters and gives some applications thereof. Also included is Isaacs' canonical lift of Brauer characters (for $p > 2$).

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ACKNOWLEDGEMENTS

The writing of this book began in Autumn 1988 as part of the project *Darstellungstheorie endlicher Gruppen und endlich-dimensionaler Algebren*, sponsored by the Deutsche Forschungsgemeinschaft (DFG). We thank the DFG for its generous support. We also thank the National Science Foundation, the Mathematical Sciences Research Institute (Berkeley) and the Ohio University Research Council for assistance. We also thank the following universities for their assistance and resources: Johannes Gutenberg Universität, Mainz (in particular, Fachbereich Mathematik), Universität Heidelberg (IWR), and Ohio University (Department of Mathematics).

We thank editors Roger Astley and David Tranah of Cambridge University Press for their assistance. Mei Lan Jin (Mathematics Typing Studio, Marion, Ohio) has done a splendid job of preparing a camera-ready manuscript using TEX. Finally, we thank numerous mathematicians whose work has influenced these pages and/or with whom we had valuable discussions.