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These notes arise from lectures presented in Florence under the auspices of the Accademia Nazionale dei Lincei and deal with an area that lies at the crossroads of mathematics and physics. The material presented here rests primarily on the pioneering work of Vaughan Jones and Edward Witten relating polynomial invariants of knots to a topological quantum field theory in  $2+1$  dimensions. Professor Atiyah here presents an introduction to Witten's ideas from the mathematical point of view. The book will be essential reading for all geometers and gauge theorists as an exposition of new and interesting ideas in a rapidly developing area.

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# *The geometry and physics of knots*

MICHAEL ATIYAH

*Master of Trinity College, Cambridge*



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## *Preface*

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These lecture notes are an expanded version of the series of lectures I gave, by invitation of the Accademia Nazionale dei Lincei, at the University of Florence in November 1988. They have also benefited from the seminar I ran in Oxford during that Autumn term. I am grateful in particular to Graeme Segal, Nigel Hitchin and Ruth Lawrence who helped me to run that seminar and to clarify many of the issues involved. I would also like to thank the mathematicians in Florence for providing such a receptive audience.

Sometimes a series of lectures may be the culmination of many years of work on a topic. In that case lecture notes may take on a definitive form, providing a careful treatment of the subject. On other occasions the lectures may come at the beginning of some new development, in which case they provide an introduction to current and future work. This is the case with these present lecture notes. The subject they deal with is just opening up and is now developing at a rapid rate. Moreover the area lies at the crossroads of mathematics and physics. This adds greatly to its interest but increases the difficulty of presentation. In due course a coherent and polished mathematical account will emerge but these lecture notes make no pretence to fulfill that role.

I have to a great extent followed the lines of the lectures as they were delivered. This means I have emphasized motivation and ideas at the expense of technicalities and formulae. As a result the reader will find no theorems even formulated,



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let alone proved, in the text. However, I have provided an extensive list of references where many of the relevant details can be found.

The material presented here rests primarily on the pioneering ideas of Vaughan Jones and Edward Witten. I have benefited greatly from extensive discussions with both of them and I hope these notes may serve a useful purpose by introducing their magnificent ideas to a wide mathematical public.

Oxford, September 1989