1

Introduction to plasticity: experimental facts

1.1 Elastic and plastic behaviours

Although the underlying microscopic mechanisms are relatively complex, viewed on a macroscopic scale – as will be the case in this book – the plastic phenomenon can be quite simple. *Plasticity*, in particular, is characterized by the existence of a stress *threshold*, or plastic threshold, and the behaviour of the medium differs, depending upon whether the stress state is on the *inside* or *right on* this threshold. On the inside of the threshold, the medium is supposed to have a linear or nonlinear *elastic* behaviour. Typically, this elastic behaviour is characterized by a stress–strain response curve of the type sketched in Fig. 1.1 for a one-dimensional model. Loading from a natural stress-free state causes a reversible increase in the measure ε of strain. The unloading path in this diagram reproduces the loading path precisely in reverse, returning to the origin as the applied stress goes back to zero. In a more vivid way, it can be said that the material possesses the 'memory' of only *one* state, the natural free state. We remind the reader that elasticity derives from an energy density $W(\boldsymbol{\varepsilon})$ per unit volume. Here we consider only small strains $\boldsymbol{\varepsilon}$. Cauchy's stress tensor is obtained by

$$\boldsymbol{\sigma} = \frac{\partial W}{\partial \boldsymbol{\varepsilon}}, \qquad W = W(\boldsymbol{\varepsilon}). \tag{1.1}$$

In the *linear anisotropic* case W is a general homogeneous function of degree 2 of the components ε_{ij} of $\boldsymbol{\varepsilon}$ in a Cartesian reference frame,

$$W = \tfrac{1}{2}\boldsymbol{\varepsilon} : \mathbf{E} : \boldsymbol{\varepsilon} = \tfrac{1}{2}\varepsilon_{ij}E_{ijkl}\varepsilon_{kl}, \tag{1.2}$$

and thus

$$\boldsymbol{\sigma} = \mathbf{E} : \boldsymbol{\varepsilon} \qquad \text{or} \qquad \sigma_{ij} = E_{ijkl}\varepsilon_{kl}, \tag{1.3}$$

where E_{ijkl} is a fourth-order tensor of elasticity coefficients. In the *isotropic* case, which is very often considered in engineering, there are only *two* independent elasticity coefficients, and eqn (1.3) takes on the special form

$$\sigma_{ij} = \lambda(\varepsilon_{kk})\delta_{ij} + 2\mu\varepsilon_{ij}, \tag{1.4}$$

where λ and μ are Lamé's elasticity coefficients.

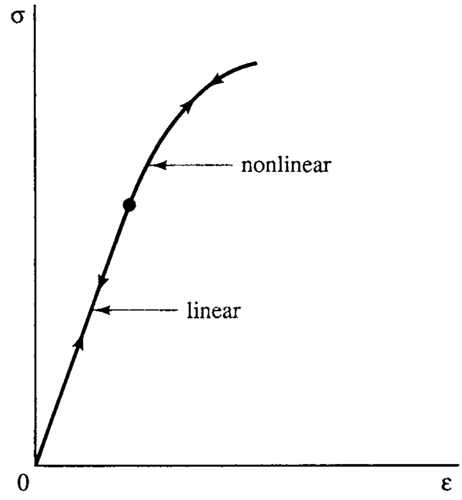
Nonlinear elasticity (curved part of the response in Fig. 1.1) may correspond to an energy W of order higher than 2 in the components of $\boldsymbol{\varepsilon}$. In general, however, the situation is more complex than that and the notion of Piola–Kirchhoff stress tensor must be introduced in the nonlinear, finite-displacement case (see Chapter 8). In the greater part of this book the elastic behaviour is described with sufficient accuracy by the simple equations (1.1)–(1.3).

Right on the plastic threshold, the mechanical behaviour of elastic–plastic materials is quite different from the elastic one. We must formulate new laws. For *perfectly plastic* media, the threshold is invariable; it is defined once and for all by the material's data and is therefore independent of the 'history' of the material. For a *not* perfectly plastic material, a material with so-called *hardening*, this threshold may evolve with the loading. This will bring some complications in the modelling.

More accurately, we should say that *we call 'plastic' the behaviour of a solid body acquiring permanent strains without cracking*, that is, without loss of the material's cohesion along certain surfaces. These permanent strains are produced starting from the plasticity threshold, or *elasticity limit*. This threshold is considered as a schematization.

An example of mechanical behaviour or response in simple traction for

Fig. 1.1. Reminder: elastic behaviour in a simple traction test



soft steel (with a low – below 0.2% – carbon content) is given in Fig. 1.2. The case of copper is also given for comparison purposes. In this figure, the plateau BC is called *the apparent yield stress limit* or *threshold*. It has been experimentally shown (experiments of P.W. Bridgman (1952) on the triaxial compression of nonporous solids and liquids) that the alteration in volume, and, consequently, the alteration of the mass per unit volume of a body, correspond to an elastic (reversible) strain, defined by an average pressure. So, in general, we leave out of consideration the insignificant alterations of the density caused by plastic strain while the *form* alteration is only due to the slip (or shear) strain. This means that, in a plastic regime, the only thing that enters into consideration is the *deviatoric* part of the strain (and, because of duality, also of the stress). We note (tr = trace)

$$\mathbf{e} = \text{deviator of } \boldsymbol{\varepsilon}, \quad e_{ij} = \varepsilon_{ij} - \tfrac{1}{3}(\mathrm{tr}\,\boldsymbol{\varepsilon})\delta_{ij}, \quad \mathrm{tr}\,\mathbf{e} \equiv 0, \tag{1.5}$$

and if τ denotes a tangential stress we consider the schematization of Fig. 1.3. Clearly, an experimentalist would describe this schematic behaviour by a succession of three 'regimes':

for $\tau < \tau_s$: $\tau = G\gamma$: this is Hooke's law in shear;

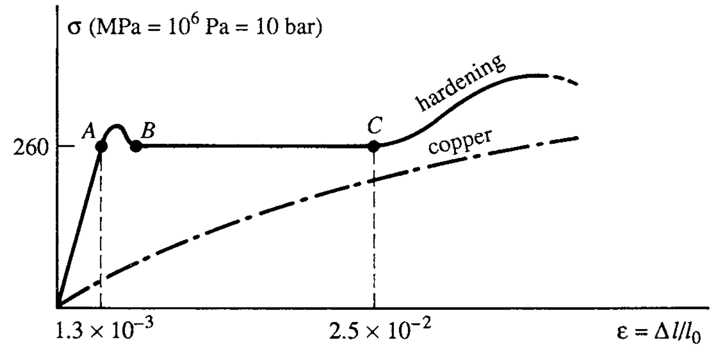
for $\gamma_e < \gamma < \gamma_s$: $\tau = \tau_s$: flow phase; the strain increases at fixed stress; it is said that the material *flows* plastically;

for $\gamma > \gamma_s$: $\tau = g(\gamma)\gamma$: hardening occurs; the *variable* quantity $g(\gamma)$ is called the *plasticity modulus*, while G was the *constant* elastic modulus; it is an experimental fact that

$$0 \leqslant g(\gamma) \leqslant G \tag{1.6}$$

Materials that exhibit a plastic regime before failure are said to be *ductile*,

Fig. 1.2. Real elastoplastic behaviour, soft steel behaviour in simple traction

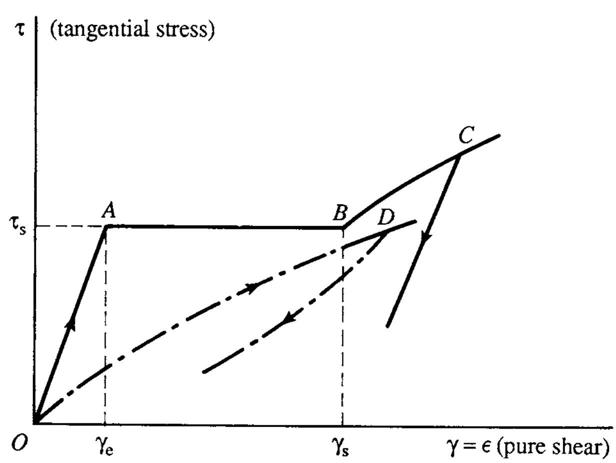


while materials that exhibit fracture while still in the elastic regime are called *brittle* materials. Obviously, the part AC in Fig. 1.3 corresponds to a *nonlinear* behaviour. In some cases the plateau AB is practically non-existent. In some other cases the straight linear-elastic part is very short or not even apparent. This is illustrated by the broken-line curve OD in Fig. 1.3 and even more vividly in Fig. 1.4 which demonstrates the typical variation in shape of a tension-test curve of an artificially prepared two-phase iron–silver material in terms of the phase concentration. Pure iron exhibits a response of the type of soft steel in Fig. 1.2, while pure silver is typically of the hardening type, like copper. But this is not all, as plasticity is essentially characterized by unloading paths that differ from the loading one.

Hardening When we unload *metallic* test samples, the return curve ABC in Fig. 1.5 is practically rectilinear. If we load again, the load curve CDE differs from the curve ABC. So, after an initial extension, the metal behaves as if it had acquired better elastic properties and a higher elastic limit, while at the same time it had lost, it is true, a great part of the plastic strain. This is the phenomenon of *work hardening*. We may also say that the point at which we stop loading defines instantaneously an elastic limit. Further loading after unloading will define a new instantaneous elastic limit and so forth.

The Bauschinger effect Work hardening is, as a rule, *oriented* in such a way that, generally speaking, the material, following a plastic strain, acquires a

Fig. 1.3. Schematic elastic–plastic behaviour

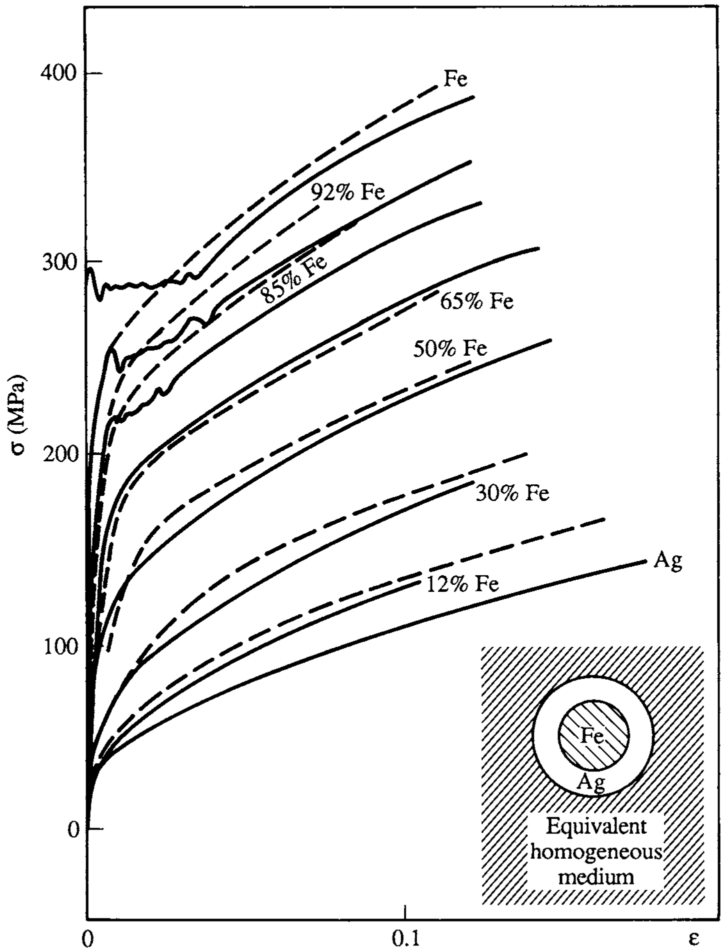


strain anisotropy; one of the manifestations of this phenomenon is the *Bauschinger effect*: a previous plastic strain with a certain sign diminishes the material's resistance with respect to the next plastic strain with the opposite sign. The plastic traction of a rod leads to a remarkable decrease of the yield limit of this rod when it is subsequently compressed again (Fig. 1.6). Here we have

$$|S_2'| < |S_1'|$$

with, in *all* cases,

Fig. 1.4. Tension-test curve of two-phase iron–silver for various phase concentrations (solid lines). Broken lines correspond to a three-phase model (after Bretheau *et al.*, 1991)



$$|S_2'| < |S_1'| + S_2 - S_1.$$

In the case of metals, $S_2' = -S_1'$ and $S_2' > -S_2$. The discovery of the effect goes back to Bauschinger (1886).

Rest and annealing With the passage of time, we can observe the partial disappearance of the hardening; this phenomenon is called the material's *rest* and it becomes increasingly apparent as the temperature goes up. In

Fig. 1.5. Hardening

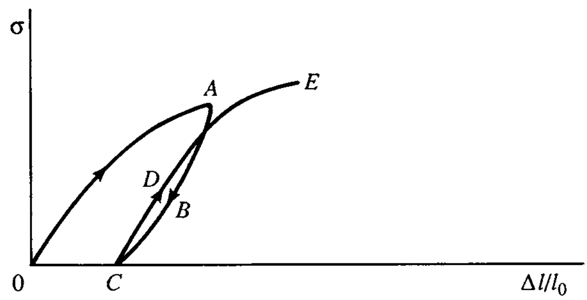
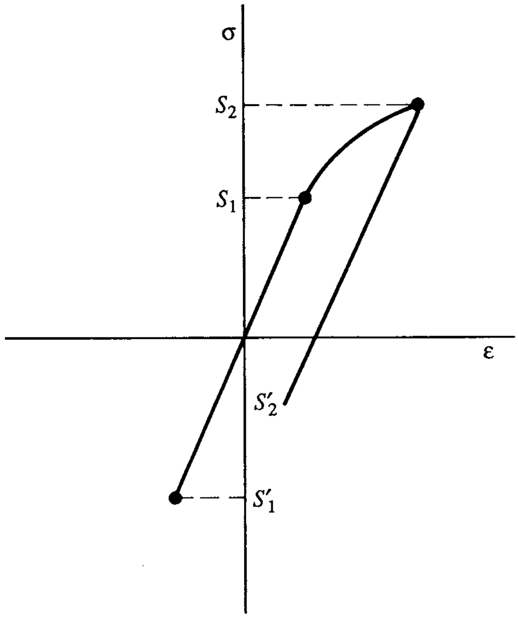


Fig. 1.6. Bauschinger effect



fact, the acquired hardening disappears under the effect of sufficiently high temperatures (we say that there is *annealing* of the material).

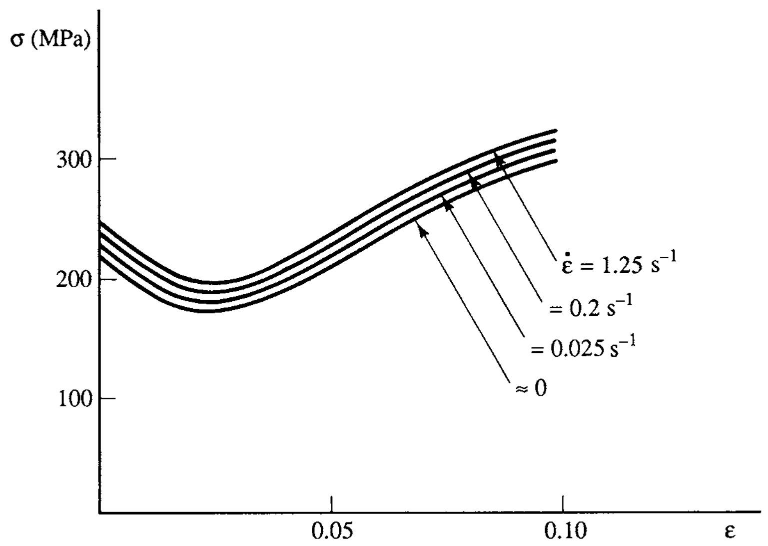
1.2 Influence of the strain rate

If the mechanical tests occur in ordinary time intervals and at room temperature, the mechanical properties of steel and of *brittle* materials in general (for example, heat-resistant materials) depend hardly at all upon the strain rate.

Fig. 1.7 illustrates the case for iron. This was well shown also in experiments by Manjoine (1944) on mild steel. Still, the velocity of the test is very important in the case of *ductile* materials (for example, lead, tin), in the case of lengthy tests on steel, copper and other metals under high-temperature conditions, and, finally, in the case of high strain rates. The effect of velocity depends to a great extent upon temperature, and, at rather low temperatures, it practically disappears. To conclude, in ordinary conditions, *the plastic strain of brittle materials is practically independent of the thermal motion of atoms and of the strain rate.* But, if velocity does play a part, then the behaviour is *viscoplastic.*

When there is neither viscosity, nor hardening, nor nonlinear elasticity, the scheme in Fig. 1.3 is reduced to *perfect elastoplasticity* (Fig. 1.8) whose rheological model is provided by a dry friction element. A word must be

Fig. 1.7. Influence of strain rate (iron)



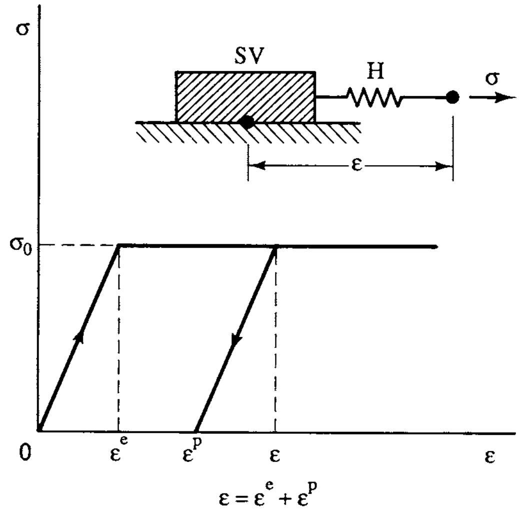
said about these rheological models. Very often we shall have recourse to them as they exhibit strong pedagogical and heuristic values. Their use in *rheology* (the science of what flows) has become general since Zener (1948). The combination in series or parallel of a few standard elements (spring, dashpot, dry friction, and others) provides a picturesque illustration of complex behaviours and the direct addition of stretches or forces, depending on the case, rapidly yields simple constitutive equations. In the present case, a spring (Hooke = H) and a dry-friction element are set in series. The *same* force is transmitted through the two elements. We call it σ. The total stretch is the sum of those in each element, i.e., $\varepsilon = \varepsilon^{e} + \varepsilon^{p}$, where $\varepsilon^{e} = \sigma/E$ if E is the spring constant. The dry-friction element slides only if a sufficiently intense force σ is transmitted to it so as to exceed, or indeed equal, the threshold of the dry-friction element σ_0. That is,

$$\left.\begin{aligned} |\sigma| \leqslant \sigma_0, &\qquad \exists \sigma = E \cdot \varepsilon^{e}, \\ \sigma = \sigma_0, &\qquad \varepsilon = \varepsilon^{e} + \varepsilon^{p} = \frac{\sigma_0}{E} + \varepsilon^{p}. \end{aligned}\right\} \tag{1.7}$$

Dry-friction elements are also called *Saint Venant* ideal (perfect) *plastic* elements (for short, SV elements). An SV element is such that

$$\varepsilon_{(SV)} = 0 \quad \text{if } \sigma < \sigma_0, \qquad \varepsilon_{(SV)} \neq 0 \text{ possibly} \quad \text{if } \sigma = \sigma_0. \tag{1.8}$$

Fig. 1.8. Perfect elastoplasticity (one dimension) rheological model

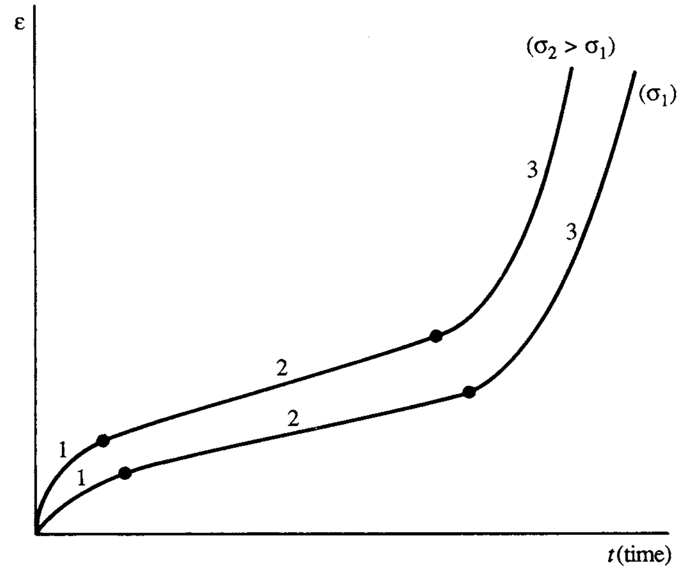


Note on creep At high temperatures we observe that plastic strain, under the effect of a relatively small stress, grows with time. This phenomenon, which is called *creep*, is expressed in certain cases by strains that increase with time, while the load remains constant, and, in some other cases, by the continuous decrease of stress, while the strain remains constant (*relaxation*). Creep is what determines the resistance and life duration of mechanical elements submitted to high temperatures. A creep curve is typically as shown in Fig. 1.9, exhibiting the regimes called primary, secondary, and tertiary creeps. In secondary creep $\dot{\varepsilon}$, the rate of strain, is nearly constant. Creep is hardly studied in this book. For this subject we refer the reader to Norton (1929) and Odqvist (1966), two pioneers in the field. Rabotnov (1969) and Cadek (1988) are also recommended references. Chapter 10 deals briefly with some aspects of creep in relation to damage (see below).

1.3 Other effects

The term *hysteresis* means that the loading and unloading curves do not coincide in spite of having similar upper and lower points in the load-versus-strain diagram. This may be due to plasticity (a phenomenon that, as we saw, is independent of velocity) or to viscoelasticity (where one or more characteristic times may interfere). We often consider a *cyclic* load

Fig. 1.9. The three regimes of creep in uniaxial constant load



(i.e., a *time-periodic loading*). In that case, either the response curve does not close up and the strain increases at each period with the so-called *ratcheting* effect, until the point of fracture at point R (Fig. 1.10(a)), or else the curve closes up after a certain number of periods in a closed cycle, called the *hysteresis cycle*. In this case we say that there is *plastic shakedown* (Fig. 1.10(b)). The presence of angular points on the cycle (Fig. 1.10(c)) hints at the existence of plastic strains that are not affected by viscosity. If there are plastic strains, they are of opposite signs on the two halves of the cycle and, after a small number of cycles, fracture occurs as a result. When there is no plastic strain, the strains during the cycle, as long as there is no progressive cracking, are viscoelastic. If, moreover, the viscosity is negligible, then we have an elastic cycle, flattened, that is (Fig. 1.10(d)); then, after a certain number of periods, the material begins to react as an elastic material: in this case we say that there is *elastic shakedown*. If the strain is imposed cyclically, there is always *plastic shakedown*.

Damage This is an alteration of the elastic properties due to the fact that, in the course of loading, the *effective resisting* area diminishes as a result of the expansion of the voids and microcracks. This phenomenon, which results in the decrease in Hooke's modulus E, may be coupled up with plasticity (see Chapter 10). Fig. 1.11 provides an example pertaining to an aluminium alloy with a slight decrease in E. This decrease is much more drastic in materials like concrete that are not our concern.

Fig. 1.10. Ratcheting and shakedown