

## Index

Bold page numbers indicate formal definitions. Greek letters are entered according to the initial letter of their name:  $\alpha$  (alpha) under 'a';  $\beta$  (beta) under 'b' etc.

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