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Ergodic theory

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Contents

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Frontmatter
More information

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For Elisabeth

Preface

Ergodic theory today is a large and rapidly developing subject. The aim of this book is to introduce the reader first to the fundamentals of the ergodic theory of point transformations and then to several advanced topics which are currently undergoing intense research. By selecting one or more of these topics to focus on, a student can quickly approach the specialized literature and indeed the frontier of the area of interest.

Of course the number of interesting topics that we have neglected is necessarily far greater than that of those we have been able to include. Thus we have to refer the reader elsewhere for discussions of, for example, operator ergodic theory, the existence of invariant measures, nonsingular transformations, orbit equivalence, differentiable dynamics, subadditive ergodic theorems, etc. Unfortunately, there do not exist coherent expositions of all of these topics; I invite those of my colleagues who are more expert than I in the areas I have omitted to do some more expository writing.

It should also be understood that, even for the advanced topics that we do discuss, their treatment here cannot be more than an entryway to the rapidly expanding specialized literature. Thus our presentations of multiple recurrence and the Ornstein theory, to mention two examples, are intended as introductions to the books of Furstenberg (1981) and Ornstein (1974), respectively.

From my point of view, ergodic theory consists of examples, convergence theorems, the study of various recurrence properties, and the theory of entropy. Each of these facets is given first a basic and later a more advanced treatment. At the introductory level, one can find the usual important basic topics: the standard examples, the mean and pointwise ergodic theorems, recurrence, ergodicity, strong mixing, weak mixing (all in Chapter 2) and the fundamentals of entropy (in Chapter 5). I have tried to make the writing as clear and complete as possible. Throughout we concentrate on single invertible measure-preserving maps on Lebesgue spaces. Some more advanced topics related to convergence theorems appear in Chapter 3, to recurrence in Chapter 4, and to entropy in Chapter 6. Chapter 3 presents a rather thorough analysis of maximal functions and their usefulness in the proof of the important convergence theorems of analysis (Lebesgue's Differentiation Theorem and the existence of the Hilbert transform), probability (the martingale convergence theorems), and ergodic theory (the Maximal Ergodic Theorem and its refinement to an equality, the Pointwise Ergodic Theorem, the Local Ergodic Theorem, the Dominated Ergodic Theorem and its converse, and the existence of the ergodic Hilbert transform). We also give Neveu's proof of the Chacon–Ornstein Theorem, as an introduction to operator ergodic theory. Chapter 4 begins with a direct construction of eigenfunctions for non-weakly-mixing transformations, which leads to a brief treatment of almost periodic functions and topological dynamics. In Section 4.3 we give an introduction to Furstenberg's approach to multiple recurrence and the Szemerédi Theorem, and Section 4.4 proves the Jewett–Krieger Theorem on the topological representation of measure-preserving transformations. The final section examines the important examples of Kakutani and Chacon, both of which are weakly but not strongly mixing and at least one of which is prime. Chapter 6 contains several nontrivial entropy computations (toral automorphisms, skew products, and induced transformations), the Shannon–McMillan–Breiman Theorem, and the fundamental facts about topological entropy. We also provide a brief introduction to Ornstein's isomorphism theory of Bernoulli shifts, proving the Ornstein Isomorphism Theorem by the Keane–Smorodinsky method, which produces a finitary (realizable by machine) map.

The background necessary to read this book is some knowledge of measure theory and functional analysis; for example, the contents of Royden's *Real Analysis* suffice. Each chapter begins with a short summary, and many sections end with exercises, which range from trivial to difficult. The list of references contains all works referred to in the text, and then some, but it is not a complete bibliography for the subject; starred entries are books and survey articles, several of which contain more extensive bibliographies. I apologize for any errors of fact, misattributions of results, misprints, passages of incompetent exposition, and all other mistakes and misjudgments; I will be happy to receive lists of errors (preferably accompanied by suggested corrections) from all who care to compile them.

Preface

This book grew out of courses given at the University of North Carolina and Yale University. During part of the writing I received financial support from the National Science Foundation, and the work was completed while I was on a leave supported by the Kenan Foundation at the Laboratoire de Calcul des Probabilités of the Université Pierre et Marie Curie (Université de Paris, VI). I thank all of those institutions for providing me with the opportunities to teach and learn this subject. I have had mathematical and editorial help especially from Brian Marcus, and also Shizuo Kakutani, Ulrich Krengel, Benjamin Weiss, Mike Keane, Shaul Foguel. Yves Derriennic, Bruce Kitchens, and Judy Halchin. My students listened patiently to my lectures, made many useful suggestions, and caught lots of errors. Janet Farrell typed the original manuscript with the assistance of Debbie Hanner Reives, Hazeline Lewis, and Doris Mahaffey. The editing and publishing were handled by David Tranah and John Samuel. My sincere thanks to all these people for their contributions.

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