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Preface

Fixed point theorems are the basic mathematical tools used in showing the existence of solution concepts in game theory and economics. While there are many excellent texts available on fixed point theory, most of them are inaccessible to a typical well-trained economist. These notes are intended to be a nonintimidating introduction to the subject of fixed point theory with particular emphasis on economic applications. While I have tried to integrate the mathematics and applications, these notes are not a comprehensive introduction to either general equilibrium theory or game theory. There are already a number of excellent texts in these areas. Debreu [1959] and Luce and Raiffa [1957] are classics. More recent texts include Hildenbrand and Kirman [1976], Ichiishi [1983], Moulin [1982] and Owen [1982]. Instead I have tried to cover material that gets left out of these texts, and to present it in such a way as to make it quickly and easily accessible to people who want to apply fixed point theorems, not refine them. I have made an effort to present useful theorems fairly early on in the text. This leads to a certain amount of compromise. In order to keep prerequisites to a minimum, the theorems are not generally stated in their most general form and the proofs presented are not necessarily the most elegant. I have tried to keep the level of mathematical sophistication on a par with, say, Rudin [1976]. In particular, only finite-dimensional spaces are used. While many of the theorems presented here are true in arbitrary locally convex spaces, no attempt has been made to cover the infinite-dimensional results. I have however deliberately tried to present proofs that generalize easily to infinite dimensional spaces whenever possible.

In an effort to show interconnections between the various results I have often given more than one proof. In fact, Chapters 9 and 21 consist largely of such interconnections. A good way to treat these chapters is as a collection of exercises with very elaborate hints. I have also tried as far as possible to indicate the sources and history of

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the various theorems. I apologize in advance for any omissions of credit or priority.

In preparing these notes I have had the benefit of the comments of my students and colleagues. I would particularly like to thank Don Brown, Tatsuro Ichiishi, Scott Johnson, Jim Jordan, Richard McKelvey, Wayne Shafer, Jim Snyder, and especially Ed Green.

I would also like to thank Linda Benjamin, Edith Huang and Carl Lydick for all their help in the physical preparation of this manuscript.

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