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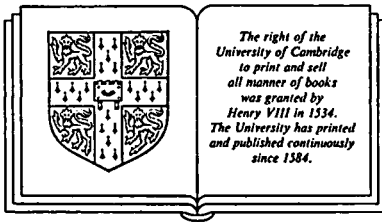
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London Mathematical Society Lecture Note Series. 144

Introduction to Uniform Spaces

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New York Port Chester Melbourne Sydney

Cambridge University Press
978-0-521-38620-3 - Introduction to Uniform Spaces
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Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011, USA
10, Stamford Road, Oakleigh, Melbourne 3166, Australia

© Cambridge University Press 1990

First published 1990

Printed in Great Britain at the University Press, Cambridge

Library of Congress cataloguing in publication data available

British Library cataloguing in publication data available

ISBN 0 521 38620 9

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Introduction

This book is based on a course of sixteen lectures recently given at the University of Oxford to an audience of undergraduate and graduate students. About half the material is classical but the remainder is not so well-known. Indeed much of it is published here for the first time.

The theory of uniform spaces was developed by Weil [25] and others in the thirties. The classic account of the subject is in Chapter II of Bourbaki [2]. However, the majority of the more recent text books on topology contain at least an outline of the theory. Moreover there are several specialized monographs, such as Isbell [7], Page [19], Roelcke and Dierolf [21], amongst others.

For students of mathematics the transition from metric spaces to general topological spaces is a major step which many find extremely difficult. Uniform spaces make an excellent intermediate stage. I have therefore written the first two chapters in such a way that they can be read by a student with no knowledge of topology. The second two chapters assume a basic knowledge of topology and are aimed at showing how the uniform world and the topological world are related.

The monographs on the theory mentioned above are written mainly with the needs of analysts in mind. Rather than go over the same ground again I have chosen to explore a different aspect of the theory. Although it has been recognized from the start that topological groups can profitably be regarded

as uniform spaces, I do not believe it has been fully appreciated that it is possible to develop a theory of uniform transformation groups. An outline of such a theory is presented here. This leads on naturally to the theory of uniform spaces over a base and hence to the theory of uniform covering spaces, in the final section.

At the end the reader will find a short bibliography and a set of over forty exercises, mainly derived from the literature on the subject but with some new problems as well. There is also an appendix containing an outline of the results concerning filters which are needed in the main text.

In writing this book I have, of course, greatly relied on Bourbaki [2]. Also I have been much influenced by the work of Roelcke and Dierolf [21]. Of the many papers I have consulted I would particularly mention those of Arens [1], Collins [4] and de Vries [24]. Finally, I would like to thank Mr. Paul Ling and Professor Don Shimamoto, who attended the lectures, for some helpful suggestions; the latter very kindly read the final draft of the text and detected various errors which had escaped my attention.

Mathematical Institute, Oxford
July 1989