# *The problem of the earth's shape From Newton to Clairaut*

This book investigates, through the problem of the earth's shape, part of the development of post-Newtonian mechanics by the Parisian scientific community during the first half of the eighteenth century. In the *Principia* Newton first raised the question of the earth's shape. John Greenberg shows how continental scholars outside France influenced efforts in Paris to solve the problem, and he also demonstrates that Parisian scholars, including Bouguer and Fontaine, did work that Alexis-Claude Clairaut used in developing his mature theory of the earth's shape. This evolution of Parisian mechanics proved not to be the replacement of a Cartesian paradigm by a Newtonian one, a replacement that might be expected from Thomas Kuhn's formulations about scientific revolutions, but a complex process instead involving many areas of research and contributions of different kinds from the entire scientific world. For example, Newtonian "normal" science does not take into account crucial developments in continental mathematics used to tackle the problem at issue.

Greenberg both explores the myriad of technical problems that underlie the historical development of part of post-Newtonian mechanics, which have been analyzed only rarely by Western scholars, and embeds his technical discussion in a framework that involves social and institutional history, politics, and biography. Instead of focusing exclusively on the historiographical problem, Greenberg shows as well that international scientific communication was as much a vital part of the scientific progress of individual nations during the first half of the eighteenth century as it is today.

# THE PROBLEM OF THE EARTH'S SHAPE FROM NEWTON TO CLAIRAUT

*The rise of mathematical science in eighteenth-century Paris and the fall of "normal" science*

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To my wife Maïté
without whose loving care, encouragement, and
help this book would never have been written

and

to my children Philippe and Sylvie

and

to the memory of my parents
Leonard S. Greenberg
(1913–1987)

and

Margaret O'Briant Greenberg
(1911–1992)

# *Contents*


| | | |
|---|---|---|
| *List of illustrations* | | *page* viii |
| *Preface* | | xv |
| 1 | Isaac Newton's theory of a flattened earth (1687, 1713, 1726) | 1 |
| 2 | The state of the problem of the earth's shape in the 1720s: Stalemate | 15 |
| 3 | The revival of geodesy in Paris (1733–1735) | 79 |
| 4 | Pierre Bouguer and the theory of homogeneous figures of equilibrium (1734) | 89 |
| 5 | Maupertuis: On the theory of the earth's shape (1734) | 107 |
| 6 | Alexis-Claude Clairaut's first theories of the earth's shape | 132 |
| 7 | Interlude I: Integral calculus (1690–1741) | 225 |
| 8 | Interlude II: The Paris Academy's contest on the tides (1740) | 400 |
| 9 | Clairaut's mature theory of the earth's shape (1741–1743): First substantial connections between the revival of mathematics in Paris and progress in mechanics there | 426 |
| 10 | Epilogue: Fontaine's and Clairaut's advances in the partial differential calculus revisited, or the virtues of interrelated developments in mathematics and science, and the fall of "normal" science | 620 |
| *Notes* | | 637 |
| *Bibliography* | | 755 |
| *Index* | | 779 |

# Illustrations

Plate 1. Alexis-Claude Clairaut (engraved by Delafosse in the style of Carmontel, Bibliothèque Nationale, Paris) *page* 133

1 Meridian of a flattened ellipsoid of revolution 3
2 Evolute of a meridian of an elongated figure of revolution (Mairan 1720: Figure 1) 19
3 Radii of curvature of a meridian of an elongated figure of revolution (Mairan 1720: Figure 4) 20
4 Meridians of an elongated figure of revolution and inscribed sphere (Mairan 1720: Figure 5) 21
5 Meridians of a flattened figure of revolution and circumscribed sphere (Mairan 1720: Figure 7) 24
6 Meridian of a nonspherical figure of revolution and plane curve from the center of the figure to the meridian which are tangent to the directions of the attraction at points on the curve (Mairan 1720: Figure 8) 29
7 Radii of curvature of a meridian of an elongated ellipsoid of revolution and plane curves that all have the same evolute as the meridian of the elongated ellipsoid of revolution (Mairan 1720: Figure 9) 31
8 Meridians of a family of concentric, elongated ellipsoids of revolution which all have the same ellipticity, hence which are geometrically similar, and the parabolas orthogonal to the members of the family (Mairan 1720: Figure 10) 32
9 Meridian of an elongated figure of revolution and its radii of curvature (Mairan 1720: Figure 12) 42
10 Meridian of a flattened figure of revolution and its radii of curvature (Mairan 1720: Figure 13) 43
11 Meridian of an ellipsoid of revolution whose ellipticity is infinitesimal (Maupertuis 1735: figure on p. 99) 81
12 Meridian of a homogeneous figure of revolution and the corresponding directrix (Bouguer 1734: Figure 1) 92
13 Application of the principle of the plumb line along a meridian of a homogeneous figure of revolution (Bouguer 1734: Figure 2) 95

14 Meridian of a homogeneous conchoid of revolution (Bouguer 1734: Figure 3) 100
15 Meridian of a homogeneous figure of revolution made up of two identical ellipsoids of revolution which intersect (Bouguer 1734: Figure 4) 101
16 Clairaut's figure used for determining the magnitude of the attraction at a pole of a homogeneous flattened ellipsoid of revolution which attracts according to the universal inverse-square law and whose ellipticity is infinitesimal (Clairaut 1809: Figure 12) 137
17 Figure used for determining the magnitude of the attraction at a pole of a homogeneous elongated ellipsoid of revolution which attracts according to the universal inverse-square law and whose ellipticity is infinitesimal 157
18 Meridian of a stratified figure whose pole is at *B* and whose polar radius is *CB* composed of an infinite number of concentric, individually homogeneous flattened ellipsoids of revolution all of which have a common axis of symmetry, whose ellipticities are all infinitesimal and equal, and which attract according to the universal inverse-square law (Clairaut 1738b: Figure 2) 182
19 Meridian of a stratified figure whose pole is at *N* and whose polar radius is *CN* (see Figure 18) composed of an infinite number of concentric, individually homogeneous ellipsoids of revolution all of which have a common axis of symmetry, whose ellipticities are all infinitesimal and equal, and which attract according to the universal inverse-square law (Clairaut 1738b: Figure 3) 185
20 Line segments in various directions at point *N* in Figure 18 in the plane of the meridian (Clairaut 1738b: Figure 5) 189
21 A family of plane curves that all lie in the same plane 267
22 The brachistochrone in a vacuum (Fontaine 1764b: "Page 22, Plate 1" located at the end) 270
23 Infinitesimal variations of the local brachistochrone in a vacuum 274
24 The brachistochrone in a vacuum (Jakob Bernoulli 1697: Table IV, Figure V I) 274
25 The tautochrone (Fontaine 1734b: figure on p. 371) 276
26 Plane curve (the tautochrone) decomposed into a family of segments 283
27 Plane curve decomposed into a family of segments 290
28 Segments of a plane curve with common endpoint (where segments are determined by initial conditions of motion along the curve) 291
29 Infinitesimally close members of a family of plane curves in the same plane whose parameter is *p* 314
30 Daniel Bernoulli's figure used for calculating the height *Bb* of the earth's neap tide in the presence of a single, attracting body at *A* (Daniel Bernoulli 1740: figure on p. 157) 404

31 Daniel Bernoulli's figure used for determining the magnitude of the attraction at the equator of a homogeneous elongated ellipsoid of revolution which attracts according to the universal inverse-square law and whose ellipticity is infinitesimal (Daniel Bernoulli 1740: figure on p. 145) 406
32 Daniel Bernoulli's figure used for determining the magnitude of the attraction at a pole of a homogeneous elongated ellipsoid of revolution which attracts according to the universal inverse-square law and whose ellipticity is infinitesimal (Daniel Bernoulli 1740: figure on p. 143) 407
33 Clairaut's figure used for calculating the magnitude of the attraction at point *M* on the surface of a homogeneous ellipsoid of revolution which attracts according to the universal inverse-square law (Clairaut 1743: figure on p. 168) 414
34 Clairaut's figure used for calculating the magnitude of the attraction at point *M* on the surface of a homogeneous ellipsoid of revolution which attracts according to the universal inverse-square law (Clairaut 1743: figure on p. 170) 415
35 Meridians of a homogeneous outer fluid layer whose surfaces are geometrically similar, flattened ellipsoids of revolution, a layer whose density is zero, and a spherical fluid core of the same density as the outer fluid layer, where the outer layer and spherical core attract according to the universal inverse-square law (Clairaut 1741c: figure on p. 365) 422
36 Figure used for showing that the principle of balanced columns and the plumb line are special cases of Clairaut's first new principle of equilibrium (Clairaut 1743: figure on p. 4) 429
37 Figure used to derive Clairaut's second new principle of equilibrium from Clairaut's first new principle of equilibrium (Clairaut 1743: figure on p. 6) 431
38 Figure used to show that Clairaut's first new principle of equilibrium follows from Clairaut's second new principle of equilibrium together with one or the other of the principles of balanced columns or the plumb line (Clairaut 1743: figure on p. 8) 434
39 Figure used to show that Clairaut's first new principle of equilibrium holds for channels at the surface of a homogeneous fluid mass whose densities are the same as the density of the mass when the principle holds for channels in the interior of the mass filled with fluid from the mass 436
40 Figure used to show that a central force whose magnitude at points in a plane through the center of force depends upon the angles made with respect to some axis of reference through the center of force in

the plane does not permit Clairaut's second new principle of equilibrium to hold (Clairaut 1743: figure on p. 29) 439

41 Figure of revolution and two channels at the surface whose endpoints are situated on the same circles perpendicular to the axis of symmetry (Clairaut 1743: figure on p. 12) 443

42 Cross sections of the level layers of a homogeneous mass of fluid (Clairaut 1743: figure on p. 41) 449

43 Figure used for deriving Clairaut's third new principle of equilibrium from Clairaut's second new principle of equilibrium 452

44 Meridian of a quadrant of a homogeneous fluid figure of revolution and interior channel expressed in two rectangular coordinates in the plane of the meridian (Clairaut 1743: figure on p. 36) 455

45 Figure used to work out a mathematical representation of Clairaut's third new principle of equilibrium for a homogeneous fluid figure of revolution (Clairaut 1743: figure on p. 44) 465

46 Meridian of a quadrant of a homogeneous fluid figure of revolution and interior channels expressed in polar coordinates in the plane of the meridian (Clairaut 1743: figure on p. 79) 473

47 Meridian of a quadrant of a homogeneous fluid figure of revolution and interior channel in the plane of the meridian which is composed of a straight portion directed toward the center of the figure and a circular portion whose center is the center of the figure (Clairaut 1743: figure on p. 89) 477

48 Figure used for determining efforts along channels when there is no symmetry (Clairaut 1743: figure on p. 98) 484

49 Figure used for showing that if a first-order "differential" equation $dz-\omega(z,x,y)dx-\theta(z,x,y)dy=0$ represents a surface, then Fontaine's "conditional equation" must hold (Clairaut 1740b: figure on p. 309) 490

50 Clairaut's figure used for showing that when Clairaut's first new principle of equilibrium holds for channels inside a stratified fluid figure whose strata have finite widths, when the channels are filled with fluid from the figure, then the surfaces between pairs of contiguous homogeneous fluid strata must all be "level surfaces" with respect to the effective gravity if the principle of the plumb line holds at the surface of the figure and conversely (Clairaut 1743: figure on p. 129). 498

51 Clairaut's figure used to show that if a homogeneous fluid figure is attracted by a central force, where the fixed center of force is situated inside the figure, then Clairaut's second new principle of equilibrium holds for all closed channels inside the figure which are filled with fluid from the figure (Clairaut 1743: figure on p. 17) 508

52 Clairaut's figure used to show that Clairaut's second new principle of equilibrium holds for all closed channels inside a homogeneous fluid

figure of revolution shaped like a ring which is attracted by multiple centers of force, when the channels are filled with fluid from the figure (Clairaut 1743: figure on p. 19) 513
53 Maupertuis's figure used to determine the meridian of a homogeneous fluid figure of revolution shaped like a ring which is attracted by multiple centers of force (Maupertuis 1732b: figure on p. 63) 515
54 Clairaut's figure used to show that if a homogeneous fluid figure of revolution is attracted in accordance with any of Bouguer's hypotheses, then the figure must have a solid nucleus in order to be a figure of equilibrium (Clairaut 1743: figure on p. 64) 523
55 Clairaut's figure used to show that if a homogeneous fluid figure of revolution is attracted in accordance with any of Bouguer's hypotheses, then the fluid in the figure can exist in a state of equilibrium whenever the magnitudes of the attraction at points in the plane of a meridian only depend upon the distances of the points from Bouguer's directrix in that plane and do not vary in any other way, whatever the shape of the directrix (Clairaut 1743: figure on p. 69) 527
56 Meridian of a quadrant of a stratified ellipsoid of revolution whose ellipticity is infinitesimal surrounded by a homogeneous fluid layer in equilibrium whose surface is also an ellipsoid of revolution whose ellipticity is infinitesimal, where the whole figure attracts according to the universal inverse-square law (Clairaut 1743: figure on p. 210) 556
57 Clairaut's figure used to illustrate his assertion that the infinite number of infinitesimally thin strata, or surfaces of constant density, of a stratified fluid figure that attracts according to the universal inverse-square law are also the "level surfaces" of the fluid figure with respect to the effective gravity (Clairaut 1743: figure on p. 266) 569
58 Clairaut's figure used to show that the homogeneous outer fluid layer of one of MacLaurin's figures that attracts according to the universal inverse-square law cannot be a figure of equilibrium (Clairaut 1742f: figure on p. 380) 586
59 Meridian of a homogeneous figure shaped like an ellipsoid of revolution which attracts in accordance with the universal inverse-square law and which revolves around its axis of symmetry (its polar axis) 638
60 Meridian of a homogeneous figure shaped like a flattened ellipsoid of revolution which attracts in accordance with the universal inverse-square law and which revolves around its axis of symmetry (its polar axis) 639
61 Meridian of a homogeneous figure shaped like a flattened ellipsoid of revolution which attracts in accordance with the universal inverse-square law, which revolves around its axis of symmetry (its polar axis), and all of whose columns from center to surface balance or weigh the same 641

62 Figure for expressing the principle of the plumb line (see Figure 13) 651
63 Figure for expressing the principle of the plumb line in rectangular coordinates 652
64 Quadrants of an ellipse and inscribed circle (Maupertuis 1734b: figure on p. 98) 659
65 Figure used for expressing the principle of the plumb line to terms of first order 670
66 Figure used to show that the principles of balanced columns and the plumb line are equivalent when Clairaut's second new principle of equilibrium holds 699
67 Clairaut's figure used to show that the total efforts produced by centrifugal force along the two plane channels *ab* and $\alpha\beta$ filled with homogeneous fluid of the same density which revolve around the axis *Pp* at the same angular speed are equal (Clairaut 1743: figure on p. 9) 700
68 Figure used to illustrate the application of the "theory of inclined planes" 701
69 Figure used to illustrate the application of the "theory of inclined planes" 701
70 Figure used to show that the total effort produced by centrifugal force around a closed plane channel filled with homogeneous fluid which revolves around an axis *Pp* is equal to zero (Clairaut 1743: figure on p. 11) 702
71 Figure used to illustrate the application of the "theory of inclined planes" 703
72 Figure used to illustrate the application of the "theory of inclined planes" 703
73 Variation of Figure 47 706
74 Figure used to demonstrate Clairaut's fundamental theorem for stratified figures of equilibrium 708
75 "$y$ = a constant" is the equation of surfaces of constant density *P* 733

# *Preface*

The story recounted in the following pages is a chapter in the history of physical sciences in France. During the period in which a large part of this story takes place, the physical sciences in France have usually been represented in ways that accord with the following scheme: the publication of Isaac Newton's *Principia* in 1687 rounds off a seventeenth-century European scientific revolution. Now, the seventeenth-century scientific revolution scarcely began with Newton, but an irreconcilable conflict between the Newtonian world system and the preceding "paradigm," the Cartesian world system, means that Newton's *Principia* was certainly a revolutionary work. Although "crucial experiments" need not enter into the genesis of a scientific revolution (Copernicus, for example, simply reinterpreted old data; it was not until much later that any new observations began to play a role in determining the course of the "Copernican Revolution"), they did function decisively in giving Newton the victory over Descartes. "Normal science" was the "mopping up" of Newton's system by the French, beginning in the 1730s and 40s. The story ends with the death of Descartes's last advocate. Cartesianism becomes a dead issue. Any reader of Kuhn (1962 or 1970) should recognize the jargon I have used here and have no trouble imagining the scheme. Moreover, the plan may well strike Kuhn's reader as thoroughly plausible. Thus before beginning my story, I offer the following words of forethought. The story may be viewed as simply depicting an inevitable mopping up of a section of Newton's *Principia* – the articulation of a "Newtonian paradigm," and its extension to cover anomalies, by means of "puzzle solving" within the "paradigm," to use Kuhn's language. But it might also involve considerations that go beyond the confines of Newtonianism alone. If so, the problem of what makes up normal science, which Kuhn contrasts with "revolutionary" science, is raised. In that case, deciding which of two interpretations of the story that follows makes the most sense requires dealing with current issues in the history and philosophy of science.

I shall not sketch here the conclusions that I have drawn from the following story. If I did I could risk reducing the story's verve. I have tried to write a narrative that keeps the reader wondering.

I will say, however, that problems of figures of equilibrium is a theme that reappears throughout the story. The story takes place in Paris during the first half of the eighteenth century. Today the investigation of problems of figures of equilibrium includes dynamical issues that were not dealt with during that time. The kinds of questions asked today involve solving functional equations and integral equations. Since the eighteenth century very powerful, highly ingenious methods have been developed to tackle these questions. Despite the invention of such mathematical tools, however, it is still not known to this day whether all figures of equilibrium of a rotating fluid mass that attracts in accordance with the universal inverse-square law of gravitation have been found. This gives an idea of just how difficult problems of equilibrium have become since the first half of the eighteenth century. The little concerning the technical aspects of problems of figures of equilibrium which appear in this story, including the advances that were made during the period that the story covers, could probably be summarized in a page or two. Even so, these few achievements were arrived at only with great effort, because the problems that were solved were still very difficult for the time. One of our aims is to understand how the problems were solved at all and the larger implications that such a success has.

Consequently, treating the questions that underlie the historiographic issue raised above necessitates that technical matters be examined in some depth and detail. However, it would be a mistake to think that the story that follows consists of nothing but the study of technical concerns. The technical discussion is embedded in a framework that involves social history, institutional history, politics, and biography that I interpret in ways that, to the best of my knowledge, cannot be found elsewhere. Remarks concerning social history, institutional history, politics, and biography are dispersed throughout the narrative, but there are also entire sections of the story that should in principle appeal to the historian who is not "internalist." The story's meaning goes beyond what could possibly emerge from a study confined solely to the technical side of the particular problems that make up the recurrent theme that I have already mentioned. If I manage to hold the reader's interest and attention, then I may succeed in transporting him to Paris as it existed during the first half of the eighteenth century.

In addition to answering this historiographic question, which involves current issues in the history and philosophy of science, the story has a moral that concerns the growth of science in a sense that differs from the one that is at issue in the historiographic problem. We shall find that international scientific communication was as much a vital part of the scientific progress of individual nations during the first half of the eighteenth century as it is today.

Research for this book was made possible by a University of Wisconsin travel grant in 1977, a National Science Foundation national needs postdoctoral fellowship in 1979–80, a Fulbright research scholarship in 1984–5, a National

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I also thank Dan Siegel, George Downs, and Bob Weinstock for providing the subsidy needed to publish this book.

I call attention to the fact that I only discovered David Beeson's *Maupertuis: An Intellectual Biography* while my own manuscript was being copy-edited and that I only discovered Rob Iliffe's article of 1993 on Maupertuis, Jesper Lützen's article of 1994 on Liouville, and John Heilbron's *Weighing the Imponderables and Other Quantitative Science around 1800* while the page proofs were undergoing final correction.

This book was written in very difficult circumstances. Since October of 1983 I have been afflicted with spastic paraparesis, an insidiously progressive form of multiple sclerosis. I have been ill the whole time that I did much of the research and all of the writing of this book. I apologize for not having included a subject index. By the time that I received proofs I was too physically handicapped to handle the preparation of a subject index for such an intricate work. I am

indebted to Benedicte Bilodeau at the Centre Alexandre Koyré, Claudine Pouret at the Archives de l'Académie Royale des Sciences, Claudine Billoux at the Archives de l'Ecole Polytechnique, Sally Grover at the Library of the Royal Society, and Fritz Nagel of the Bernoulli Edition for sending me material when I was no longer able to travel.

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