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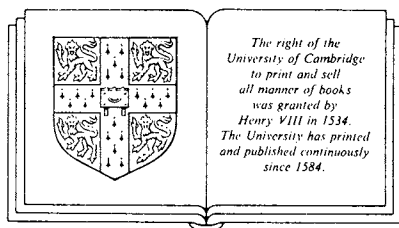
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To Birthe, Hanne, Helle and Martin.

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PREFACE

The theories of braids and covering maps offer many suitable topics for a second course in topology accessible to beginning graduate students and still leading to current research. This book is an outgrowth of such a topics course given at the University of Maryland, College Park, in the fall term 1986.

The book has four chapters. The choice of material in the first two chapters, on braid groups with applications to the study of links, has been strongly influenced by the book of Joan Birman: *Braids, Links and Mapping Class Groups*. It will be apparent that I am much indebted to that book. The material in the final two chapters is, to a large extent, based on my own research on polynomial covering maps. From an algebraic point of view, an n -fold polynomial covering map can be viewed as a homomorphism of the fundamental group of the base space into the Artin braid group on n strings. This is the link between the two themes in the book.

Prerequisites for the book include basic courses in topology, in particular the fundamental group and covering maps, algebra and analysis. It is possible to fill in missing prerequisites along the way, since the concepts treated are very basic and provide good introductions to the relevant subjects. Results from algebraic topology not directly referenced can all be found in the books of Spanier: *Algebraic Topology*, or Steenrod: *The Topology of Fibre Bundles*.

It is a pleasure to thank Lars Gæde not only for many discussions during the course of writing but also for contributing Appendix 1 containing a presentation for the coloured (pure) braid group. I am likewise indebted to Hugh Morton for allowing me to reprint his paper "Threading knot diagrams", *Math. Proc. Camb. Phil. Soc.* 99(1986), 247–260, in Appendix 2. Also thanks to Jesper Michael Møller who read the entire manuscript and suggested many improvements.

Helle Wolter performed a skilful and efficient typing job and Beth Beyerholm made most of the figures.

Finally, I want to extend my gratitude to the students in my class at College Park, who made it such a delight to present these lectures.

Lyngby, August 1989

Vagn Lundsgaard Hansen.

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