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978-0-521-38165-9 - Matroid Applications
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This volume, the third in a sequence that began with *The Theory of Matroids* and *Combinatorial Geometries*, concentrates on the applications of matroid theory to a variety of topics from engineering (rigidity and scene analysis), combinatorics (graphs, lattices, codes and designs), topology and operations research (the greedy algorithm). As with its predecessors, the contributors to this volume have written their articles to form a cohesive account so that the result is a volume that will be a valuable reference for research workers.

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ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521381659

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First published 1992

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data

- Matroid applications / edited by Neil White.
p. cm. — (Encyclopedia of mathematics and its applications ; v. 40)
Includes bibliographical references and index.
ISBN 0-521-38165-7
1. Matroids. I. White, Neil. II. Series.
QA166.6.M38 1992
511'.6—dc20 91-27184 CIP

ISBN 978-0-521-38165-9 hardback

Transferred to digital printing 2007

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P R E F A C E

This is the third volume of a series that began with *Theory of Matroids* and continued with *Combinatorial Geometries*. These three volumes are the culmination of more than a decade of effort on the part of the many contributors, potential contributors, referees, the publisher, and numerous other interested parties, to all of whom I am deeply grateful. To all those who waited, please accept my apologies. I trust that this volume will be found to have been worth the wait.

This volume begins with Walter Whiteley's chapter on the applications of matroid theory to the rigidity of frameworks: matroid constructions prove to be rather useful and matroid terminology provides a helpful language for the basic results of this theory. Next we have Deza's chapter on the beautiful applications of matroid theory to a special aspect of combinatorial designs, namely perfect matroid designs. In Chapter 3, Oxley considers ways of generalizing the matroid axioms to infinite ground sets, and Simões-Pereira's chapter on matroidal families of graphs discusses other ways of defining a matroid on the edge set of a graph than the usual graphic matroid method. Next, Rival and Stanford consider two questions on partition lattices. These lattices are a special case of geometric lattices and the inclusion of this chapter will provide a lattice-theoretic perspective which has been lacking in much current matroid research (but which seems alive and well in *oriented* matroids). Then we have the comprehensive survey by Brylawski and Oxley of the Tutte polynomial and Tutte–Grothendieck invariants. These express the deletion–contraction decomposition that is so important within matroid theory and some of its important applications, namely graph theory and coding theory. Björner describes the homology and shellability properties of several simplicial complexes associated with a matroid; the complexes of independent sets, of broken circuits, and of chains in the geometric lattice. This chapter and the previous one constitute a study of the deepest known matroid

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Preface

invariants. We conclude with an exposition by Björner and Ziegler of greedoids, a generalization of matroids that embody the greedy algorithm and hence are very useful in operations research.

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