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Edited by A. Baker, B. Bollobas and A. Hajnal
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A Tribute to Paul Erdős

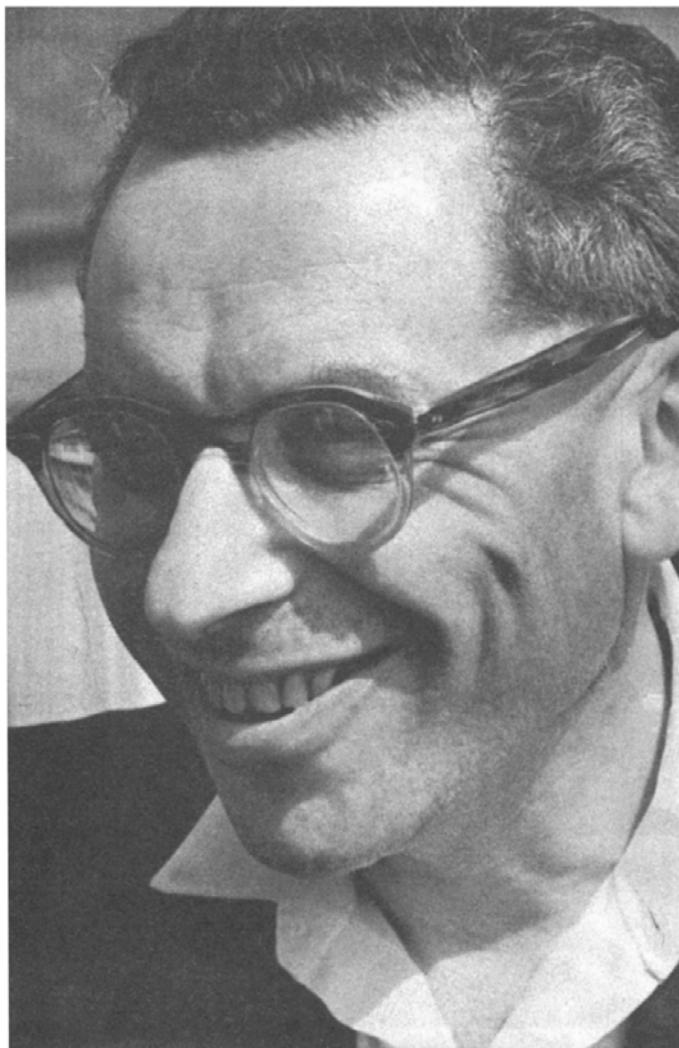
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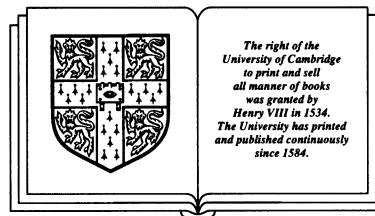
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Edited by
A. Baker
Trinity College, Cambridge
B. Bollobás
Trinity College, Cambridge
A. Hajnal
Hungarian Academy of Sciences



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Preface

This volume is dedicated to Paul Erdős, who has profoundly influenced mathematics this century. He has worked in number theory, complex analysis, probability theory, geometry, interpolation theory, algebra, set theory and, perhaps above all, in combinatorics. His theorems and conjectures have had a decisive impact. In particular, he, more than anybody else, is the founder of modern combinatorics, he pioneered probabilistic number theory, he is the master of random methods in analysis and combinatorics, and he has created the fields of Ramsey theory and the partition calculus of set theory.

Paul Erdős is the consummate problem solver: his hallmark is the succinct and clever argument, often leading to a solution from ‘the book’. He loves areas of mathematics which do not require an excessive amount of technical knowledge but give scope for ingenuity and surprise. The mathematics of Paul Erdős is the mathematics of beauty and insight.

One of the most attractive ways in which Paul Erdős has influenced mathematics is through a host of stimulating problems and conjectures, to many of which he has attached money prizes, in accordance with their notoriety. He often says that he could not pay up if all his problems were solved at once, but neither could the strongest bank if all its customers withdrew their money at the same time. And the latter is far more likely.

Ever since he was greeted at the station in Cambridge by Davenport and Rado in October 1934, on his way from Hungary to Manchester to work with Mordell, Paul Erdős has travelled the world constantly: he is the archetypal peripatetic mathematician. As the ‘Professor of the Universe’, he travels light, saying that ‘private property is a nuisance’; wherever he arrives his ‘brain is open’ for new problems and ideas: ‘another roof, another proof’. He is fond of idiosyncratic expressions (bosses, slaves and

epsilons, captured and liberated, Sam and Joe, preaching, poison, cured of incurable disease), but the light-hearted, easy-going exterior hides a thoroughly professional approach to mathematics.

He has written more than 1200 papers with an astonishing number of co-authors – about 300. This phenomenon has given rise to the notion of an ‘Erdős number’: even Einstein has an Erdős number, namely 2, and the largest Erdős number is believed to be 7.

This volume consists of research papers from diverse areas of mathematics, with a spread reflecting the wide range of Paul Erdős’s interests. We are delighted to present it to him for his seventy-fifth birthday, on behalf of his many friends, collaborators and admirers, in appreciation of all that he has done for mathematics and the mathematical community during his long and fruitful career.

Alan Baker
Béla Bollobás
András Hajnal

Cambridge and Budapest

Contributors

Miklós Ajtai	IBM San Jose Research Laboratory, San Jose, California, USA
N. Alon	Department of Mathematics, Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv, Israel
James E. Baumgartner	Department of Mathematics, Dartmouth College, Hanover, New Hampshire 03755, USA
Józef Beck	Department of Mathematics, Hill Center, Rutgers University, New Brunswick, New Jersey 08903, USA, and Mathematical Institute of the Hungarian Academy of Sciences, Reáltanoda utca 13–15, H-1053 Budapest, Hungary
Béla Bollobás	Department of Mathematics and Mathematical Statistics, University of Cambridge, 16 Mill Lane, Cambridge CB2 1SB, UK
J. Bourgain	Institut des Hautes Étude Scientifiques, 35, Route de Chartres, 91440 Bures-sur-Yvette, France
F. R. K. Chung	Bell Communications Research, Morristown, New Jersey 07960, USA

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Contributors

Jacques Dixmier	11bis rue du Val de Grâce, F-75005 Paris, France
Alan Dow	Department of Mathematics, York University, North York, Ontario, Canada M3J 1P3
R. Durrett	Department of Mathematics, White Hall, Cornell University, Ithaca, New York 14853, USA
P. D. T. A. Elliott	Department of Mathematics, University of Colorado at Boulder, Boulder, Colorado 80309- 0426, USA
Paul Erdős	Mathematical Institute of the Hungarian Academy of Sciences, Reáltanoda utca 13–15, H-1053 Budapest, Hungary
T. I. Fenner	Department of Computer Science, Birkbeck College, Malet Street, London WC1E 7HX, UK
Matthew Foreman	Department of Mathematics, Ohio State University, 231 West 18th Avenue, Columbus, Ohio 43210, USA
A. M. Frieze	Department of Mathematics, Carnegie Mellon University, Pittsburg, Pennsylvania 15213-3890, USA
Fred Galvin	Department of Mathematics, University of Kansas, Lawrence, Kansas 66045-2142, USA
R. L. Graham	AT&T Bell Laboratories, Murray Hill, New Jersey 07974, USA
K. Győry	Mathematical Institute, University of Debrecen, PO Box 12, H-4010 Debrecen, Hungary
Roland Häggkvist	Mathematiska Institutionen, Stockholms Universitet, Box 6701, 113 85 Stockholm, Sweden
A. Hajnal	Mathematical Institute of the Hungarian Academy of Sciences, Reáltanoda utca 13–15, H-1053 Budapest, Hungary

Contributors

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R. R. Hall	Department of Mathematics, University of York, Heslington, York YO1 5DD, UK
W. K. Hayman	Department of Mathematics, University of York, Heslington, York YO1 5DD, UK
D. R. Heath-Brown	Magdalen College, Oxford OX1 4AU, UK
Thomas Jech	Department of Mathematics, Pennsylvania State University, 215 McAllister Building, University Park, Pennsylvania 16802, USA
I. Juhász	Mathematical Institute of the Hungarian Academy of Sciences, Reáltanoda utca 13–15, H-1053 Budapest, Hungary
H. Kesten	Department of Mathematics, White Hall, Cornell University, Ithaca, New York 14853, USA
D. J. Kleitman	Department of Mathematics, M.I.T., Cambridge, MA 02139, USA
P. Komjáth	Department of Computer Science, Eötvös Loránd University, Múzeum hőrút 6–8, H-1088 Budapest, Hungary
János Komlós	Department of Mathematics, Hill Center, Rutgers University, New Brunswick, New Jersey 08903, USA
K. Kunen	Department of Mathematics, University of Wisconsin, 480 Lincoln Drive, Madison, Wisconsin 53706, USA
Attila Máté	Department of Mathematics, Brooklyn College of CUNY, Bedford Ave. and Ave. H, Brooklyn, New York 11210, USA
E. C. Milner	Department of Mathematics and Statistics, University of Calgary, 2500 University Drive NW, Calgary, Alberta, Canada T2N 1N4
H. L. Montgomery	Department of Mathematics, University of Michigan, Ann Arbor, Michigan, USA

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Contributors

Z. Nagy	Mathematical Institute of the Hungarian Academy of Sciences, Reáltanoda utca 13–15, H-1053 Budapest, Hungary
Jean-Louis Nicolas	Département de Mathématiques, Université Claude Bernard (Lyon I), F-69622 Villeurbanne Cedex, France
Paul Nevai	Department of Mathematics, Ohio State University, 231 West 18th Avenue, Columbus, Ohio 43210 USA
P. Révész	Institut für Statistik und Warscheinlichkeitstheorie, Technische Universität Wien, Wiedner Hauptstrasse 8–10/107, A-1040 Wien, Austria
A. Schinzel	Instytut Matematyczny PAN, Skr. pocztowa 137, 00-950 Warszawa, Poland
Wolfgang M. Schmidt	Department of Mathematics, University of Colorado at Boulder, Boulder, Colorado 80309-0426, USA
Saharon Shelah	Department of Mathematics, The Hebrew University, Jerusalem, Israel, and Department of Mathematics, Hill Center, Rutgers University, New Brunswick, New Jersey 08903, USA
T. N. Shorey	School of Mathematics, Tata institute for Fundamental Research, Homi Bhabha Road, Bombay 400 005, India
L. Soukup	Mathematical Institute of the Hungarian Academy of Sciences, Reáltanoda utca 13–15, H-1053 Budapest, Hungary
Joel Spencer	Courant Institute of Mathematical Sciences, 251 mercer Street, New York, New York 10012, USA
J. Szabados	Mathematical Institute of the Hungarian Academy of Sciences, Reáltanoda utca 13–15, H-1053 Budapest, Hungary

Contributors

xv

Endre Szemerédi	Department of Mathematics, Hill Center, Rutgers University, New Brunswick, New Jersey 08903, USA
Z. Szentmiklóssy	Mathematical Institute of the Hungarian Academy of Sciences, Reáltanoda utca 13–15, H-1053 Budapest, Hungary
Gérald Tenenbaum	Département de Mathématiques, Université de Nancy I, BP 239-54506 Vandœuvre les Nancy Cedex, France
A. R. Thatcher	129 Thetford Road, New Malden, Surrey KT3 5DS, UK
Carsten Thomassen	Mathematical Institute, The Technical University of Denmark, Building 303, DK-2800, Lyngby, Denmark
R. Tijdeman	Mathematical Institute, RU Leiden, Postbus 9512, 2300 RA Leiden, The Netherlands
R. C. Vaughan	Department of Mathematics, Imperial College of Science and Technology, Queen's Gate, London SW7 2BZ, UK
A. K. Varma	Department of Mathematics, University of Florida, Gainesville, Florida 32611, USA
Péter Vértesi	Mathematical Institute of the Hungarian Academy of Sciences, Reáltanoda utca 13–15, H-1053 Budapest, Hungary
W. Weiss	Mathematics Department, University of Toronto, Toronto, Canada

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