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The Technique of Pseudodifferential Operators

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To my 6 children, Stefan and Susan

Sabine and Art, Eva and Sam

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P R E F A C E

It is generally well known that the Fourier-Laplace transform converts a linear constant coefficient PDE P(D)u=f on $\mathbb{R}^n$ to an equation $P(\xi)u^{\sim}(\xi)=f^{\sim}(\xi)$, for the transforms $u^{\sim}$, $f^{\sim}$ of u and f, so that solving P(D)u=f just amounts to division by the polynomial $P(\xi)$. The practical application was suspect, and ill understood, however, until theory of distributions provided a basis for a logically consistent theory. Thereafter it became the Fourier-Laplace method for solving initial-boundary problems for standard PDE. We recall these facts in some detail in sec's 1-4 of ch.0.

The technique of pseudodifferential operator extends the Fourier-Laplace method to cover PDE with variable coefficients, and to apply to more general compact and noncompact domains or manifolds with boundary. Concepts remain simple, but, as a rule, integrals are divergent and infinite sums do not converge, forcing lengthy, often endlessly repetitive, discussions of 'finite parts' (a type of divergent oscillatory integral existing as distribution integral) and asymptotic sums (modulo order $-\infty$).

Of course, pseudodifferential operators (abbreviated ψdo's) are (generate) abstract linear operators between Hilbert or Banach spaces, and our results amount to 'well-posedness' (or normal solvability) of certain such abstract linear operators. Accordingly both, the Fourier-Laplace method and theory of ψdo's, must be seen in the context of modern operator theory.

To this author it always was most fascinating that the same type of results (as offered by elliptic theory of ψdo;'s) may be obtained by studying certain examples of Banach algebras of linear operators. The symbol of a ψdo has its abstract meaning as Gelfand function of the coset modulo compact operators of the abstract operator in the algebra.

On the other hand, hyperbolic theory, generally dealing with a group exp(Kt) (or an evolution operator U(t)) also has its manifestation with respect to such operator algebras: conjugation with exp(Kt) amounts to an automorphism of the operator algebra, and of the quotient algebra. It generates a flow in the symbol space essentially the characteristic flow of singularities. In [C_1], [C_2] we were going into details discussing this abstract approach.

We believe to have demonstrated that ψdo's are not necessary to understand these fact. But the technique of ψdo's, in spite of its endless formalisms (as a rule integrals are always 'distribution integrals', and infinite series are asymptotically convergent, not convergent), still provides a strongly simplifying principle, once the technique is mastered. Thus our present discussion of this technique may be justified.

On the other hand, our hyperbolic discussions focus on invariance of ψdo-algebras under conjugation with evolution operators, and do not touch the type of oscillatory integral and further discussions needed to reveal the structure of such evolution operators as Fourier integral operators. In terms of Quantum mechanics we prefer the Heisenberg representation, not the Schroedinger representation.

In particular this leads us into a discussion of the Dirac equation and its invariant algebra, in chapter X. We propose it as algebra of observables.

The basis for this volume is (i) a set of notes of lectures given at Berkeley in 1974-80 (chapters I-IV) published as preprint at U. of Bonn, and (ii) a set of notes on a seminar given in 1984 also at Berkeley (chapters VI-IX). The first covers elliptic (and parabolic) theory, the second hyperbolic theory. One might say that we have tried an old fashiened PDE lecture in modern style.

In our experience a newcomer will have to reinvent the theory before he can feel at home with it. Accordingly, we did not try to push generality to its limits. Rather, we tend to focus on the simplest nontrivial case, leaving generalizations to the reader. In that respect, perhaps we should mention the problems (partly of research level) in chapters I-IV, pointing to manifolds with conical tips or cylindrical ends, where the 'Fredholm-significant symbol' becomes operator-valued.

The material has been with the author for a long time, and was subject of many discussions with students and collaborators. Especially we are indebted to R. McOwen, A.Erkip, H. Sohrab, E. Schrohe, in chronological order. We are grateful to Cambridge University Press for its patience, waiting for the manuscript.

Berkeley, November 1993 Heinz O. Cordes