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978-0-521-37289-3 - Volterra Integral and Functional Equations

G. Gripenberg, S.-O. Londen and O. Staffans

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