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978-0-521-37289-3 - Volterra Integral and Functional Equations

G. Gripenberg, S.-O. Londen and O. Staffans

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Numbers following entries are page numbers; numbers in brackets refer to the theorem, lemma, etc. where the index word appears. Boldface type indicates that the entry is defined or explained.

Observe that a large number of the entries are grouped under certain key words, such as *differential resolvent*, *kernel*, *resolvent*, *solution of a (non)linear equation*, and *spectrum*.

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