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SHUI-NEE CHOW
Georgia Institute of Technology

CHENGZHI LI
Peking University

DUO WANG
Tsinghua University

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Preface

The theory of bifurcation of vector fields is the study of a family of equations that are close to a given equation. For example, the family of equations could be a system of vector fields depending on several parameters. An important problem is to understand how the topological structure of the flow generated by the family of vector fields changes qualitatively as parameters are varied. The main purpose of this book is to present some methods and results of the theory of bifurcations of planar vector fields.

Since simplifying equations is often a necessary first step in many bifurcation problems, we introduce the theory of center manifolds and the theory of normal forms. Center manifold theory is important for the reduction of equations to ones of lower dimension, and normal-form theory gives a tool for simplifying the forms of equations to the ones with the simplest possible higher-order terms near their equilibria. We introduce versal deformations of vector fields and define the codimension of a bifurcation of vector fields. This is illustrated by saddle-node and Hopf bifurcations. We discuss in detail all known codimension-two bifurcations of planar vector fields. Some special cases of higher-codimension bifurcations are also considered.

In Chapter 1, we introduce briefly the basic concepts of center manifolds. We show the existence, uniqueness, and smoothness of global center manifolds. The existence, asymptotic behavior, and foliation of local center manifolds are also discussed.

In Chapter 2, we present the theory of normal forms. We first discuss in detail normal forms of vector fields near their equilibria. We introduce two methods for computing normal forms: the matrix representation method and the method of adjoints. We also introduce normal forms of equations with periodic coefficients or with symmetries. Normal forms of diffeomorphisms and Hamiltonian systems are discussed.

Complete proofs of Poincaré and Siegel linearization theorems are presented. Takens's Theorem gives a relation between diffeomorphisms near fixed points and the time-one maps of flows of vector fields near equilibria. We introduce also versal deformations of matrices and of infinitesimally symplectic matrices and normal forms of vector fields of codimension one and two.

In Chapters 3, 4, and 5, we discuss bifurcation problems of vector fields with some degeneracies. We assume that the problems to be considered are restricted to local center manifolds and are in their normal forms up to some order. In Chapter 3, we introduce the concepts of versal deformations and the codimension of a bifurcation of vector fields. Bifurcations of codimension one near singularities and homoclinic orbits are considered. In Chapter 4, we deal with bifurcations of codimension two. For vector fields whose linear parts have double zero eigenvalues, we consider a nonsymmetrical case and the cases with $1:q$ symmetrices ($q = 2, 3, 4$ and $q \geq 5$). The case of $1:4$ symmetry is the most difficult and is far from being solved completely. For the cases in which the linear parts have one zero and one pair of purely imaginary eigenvalues, or two pairs of purely imaginary eigenvalues, we reduce them to planar systems and then give complete bifurcation diagrams. In Chapter 5, we discuss higher-codimension bifurcation problems, including Hopf and homoclinic bifurcations with any codimension and cusp bifurcations with codimension three and four.

In the last section of each chapter we give briefly the history and literature of material covered in the chapter. We have tried to make our references as complete as possible. However, we are sure that many are missing.

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