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The Geometry of Jet Bundles

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Contents

Introduction

This book is intended as an introduction to the language of jet bundles, for the reader who is interested in mathematical physics, and who has a knowledge of modern differential geometry.

Several ways of applying geometric techniques to physics are now well established in the literature: two major examples are the study of tangent and cotangent bundles in mechanics, and the use of connections on principal fibre bundles in field theories. More recently, the language of jets has appeared as a concise way of describing phenomena associated with the derivatives of maps, particularly those associated with the calculus of variations. In fact, a jet is no more than a generalisation of a tangent vector, and the geometrical theory of jet bundles includes the theories mentioned earlier as special cases. Generalisation, of course, sometimes introduces complexity: for instance, the coordinate representation used for jets bears some resemblance to the traditional coordinate representation used in the tensor calculus, but differs in that the transformation rules are no longer linear. In addition, many of the coordinate formulæ are symmetric in their indices, as a consequence of the commutativity of repeated partial differentiation, and this also introduces a certain complexity. On the other hand, the geometric nature of the theory introduces simplicity: there is, for instance, a clear geometric interpretation of the reason why the curvature of a connection is the obstruction to the integrability of the system of partial differential equations represented by the connection.

This book introduces those aspects of the theory of jet bundles which explain these local phenomena, although the theory itself is described in global terms. The first part of the book, comprising Chapters 1–3, sets out those elements of the theory of bundles and of linear structures which will be needed in subsequent chapters. Some of this material may be familiar to readers who are already acquainted with fibre bundles, although the perspective adopted here is one which ignores the existence of the structure group of the bundle.

The remainder of the book introduces the theory of jets. This is done in four distinct stages to make the task more manageable, although at a risk of some repetition. The basic definitions are given in Chapter 4, which describes first-order jets; the fundamental idea of prolongation also appears here, and is used in the specification of variational problems. Chapter 5, on second-order jets, introduces the idea of integrability, and also forms the setting for an intrinsic version of the Euler-Lagrange equations, constructed with the aid of a Cartan form. Higher-order jets are considered in Chapter 6, and a multi-index notation is adopted to deal with them; the global construction of a higher-order Cartan form also appears in this chapter. Finally, Chapter 7 uses the theory of calculus in infinite-dimensional Fréchet space to define infinite jets, and in this context proves the local exactness of the variational bi-complex; a consequence of this result is the Helmholtz condition in the inverse problem of the calculus of variations.

I should like to express my gratitude to colleagues with whom I have discussed this subject over the past few years. In particular, I should like to thank Mike Crampin, for his advice and encouragement, and Frans Cantrijn, who has read most of the manuscript and made many helpful suggestions. I am also indebted to the Research Advisory staff of the Open University's Academic Computing Service for their advice on the use of LaTeX.

D. J. Saunders
September 1988

Conventions

In this book, we suppose that all manifolds are real, and that manifolds and maps are smooth (that is to say, of class C^∞). We shall require the topology on each manifold to be Hausdorff, second-countable and connected. We shall assume, except in Chapter 7, that all our manifolds are finite-dimensional; it follows from these assumptions that our manifolds admit partitions of unity.

When using wedge products of cotangent vectors or of differential forms, it is always necessary to adopt a convention concerning the numerical factor to be employed: our convention will be such that, if α and β are cotangent vectors (or 1-forms), we may write

$$\alpha \wedge \beta = \alpha \otimes \beta - \beta \otimes \alpha$$

without any numerical factor.