

Cambridge University Press

0521368782 - The Kinematics of Mixing: Stretching, Chaos, and Transport

J. M. Ottino

Index

[More information](#)


---



---

## *Author index*

---

Italic numbers refer to figures.

- Abraham, R. H., 109, 124, 127, 152  
 Acrivos, A., 218, 299, 301, 303–7, 315  
 Allègre, C. J., 8  
 Andronov, A. A., 128  
 Aref, H., 1, 3, 130, 140, 153, 171, 216, 272,  
     316  
 Aris, R., 27, 40, 339  
 Arnold, V. I., 18, 101–2, 127, 132, 136, 146,  
     152–3  
 Ashurst, W. T., 296, 298  
 Astarita, G., 67  
 Balachandar, S., 216  
 Balazs, N. L., 150–1 and 216 [in Berry  
     *et al.* (1979)]  
 Ballal, B. Y., 218  
 Barthès-Biesel, D., 301, 304, 306  
 Batchelor, G. K., 6, 9, 52, 57–8, 62, 75,  
     139–40, 277, 297, 315–17  
 Bellinger, J. C., 317  
 Bentley, B. J., 41, 299, 302–3, 304, 307,  
     315, 317  
 Berker, R., 95  
 Berman, N. S., 318  
 Berry, M. V., 150–1, 216, 308  
 Bigg, D., 218  
 Bird, R. B., 6, 22, 316  
 Birkhoff, G. D., 145–6, 152  
 Bowen, R. M., 22, 41  
 Brenner, H., 47  
 Broadwell, J. E., 288  
 Brothman, A., 3  
 Brown, G. L., 8  
 Buckmaster, J. D., 304, 307  
 Buffham, B. A., 16, 317  
 Burgers, J. M., 57  
 Cantwell, B. J., 62  
 Caswell, J. B., 34  
 Chadwick, P., 20, 40, 62–3, 339  
 Chaiken, J., 9, 11, 216, 218  
 Chan, K. L., 6  
 Chan, W. C., 315  
 Chance, B., 17  
 Chang, H.-N., 218  
 Chella, R., 7, 72, 81, 86, 90, 94, 280–1, 285,  
     315–16  
 Chevray, R., 9, 11, 216, 218 [in Chaiken  
     *et al.* (1986)]  
 Chien, W.-L., 9, 201, 213, 216  
 Chong, M. S., 262, 266–7, 272  
 Cohen, E. G. D., [see Helleman (1980) and  
     Swinney (1985)]  
 Coleman, B. D., 94–5, 339  
 Coles, D., 62  
 Cooper, A. R., 16  
 Corcos, G. M., 290, 291–3, 292, 295  
 Corrsin, S., 4, 6, 9, 11, 316  
 Cox, R. G., 304, 317  
 Crawford, J. D., 271  
 Damköhler, G., 316  
 Danckwerts, P. V., 4, 7, 16, 187, 317  
 Danielson, T. J., 271  
 Davies R. M. [see Batchelor and Townsend  
     (1956)]  
     de Gennes, P. G., 318  
 Devaney, R. L., 127  
 Dimotakis, P. E. Fig. 1.3.5 (see color plates),  
     12, 13, 60  
 Doherty, M. F., 150, 173  
 Dombre, T., 221, 227  
 Duffing, G., 218  
 Eckart, C., xi, 4  
 Eringen, A. C. [see Bowen (1976)]  
 Escande, D. F., 153<sub>14</sub>  
 Fairlie, B. D., 35, 272  
 Feldman, S. M., 3  
 Feigenbaum, M. J., 127  
 Feingold, M., 272  
 Fields, S. D., 13, 285, 286–7, 289  
 Finlayson, B. A., 224  
 Fisher, D. A., 315  
 Foister, R. T., 316

- Franjione, J. G., 219, 227, 228–35, 238–45, 271, 294
- Friedrichs, K. O., 22
- Frisch, U., 221 and 271 [in Dombre *et al.* (1986)], 272
- Galloway, D., 272
- García-Rejón, A. [see Khakhar, Chella, and Ottino (1984) and Leal (1984)]
- Gibbs, J. W., 191, 217
- Gibson, C. H., 288, 290, 296 [in Ashurst *et al.* (1986)], 298 [in Ashurst *et al.* (1987)]
- Giesekus, H., 41
- Gillani, N. W., 250
- Gogos, C. G., 16
- Goldberg, E. D., 16
- Goldstein, G. A., 272
- Goldstein, H., 152–3
- Gollub, J. P., 216
- Grace, H. P., 315, 317
- Greene, J. M., 128, 153, 221 and 271 [in Dombre *et al.* (1986)]
- Guckenheimer, J., 97, 101, 112, 127–9, 141, 152–3, 162, 177, 266
- Hama, F. R., 25, 62
- Hawthorne, W. R., 4, 5
- Helleman, R. H. G., 132, 145, 152
- Heller, J. P., 40
- Hénon, M., 220, 221 and 271 [in Dombre *et al.* (1986)], 269
- Hinch, E. J., 305, 307
- Hirsch, M. W., 25, 97, 101–3, 127–8, 136
- Hoffman, N. R. A., 10, 216
- Holland, W. R., 16
- Holloway, G., xi
- Holmes, P., 97, 101, 112, 127–9, 141, 152–3, 162, 177, 266
- Hottel, H. C., 4, 5
- Huijgol, R. R., 34, 67
- Hyman, D., xi
- Ioos, G., 127
- Jeffrey, G. B., 218
- Jepson, C. H., 4, 7, 16
- Jones, S. W., 153, 272
- Joseph, D. D., 127
- Jou, W.-H., 295 [in McMurtry *et al.* (1986)]
- Kadanoff, L. P., 272
- Kanterovich, L. V., 224
- Kapila, A. K., 317
- Karam, H. J., 317
- Karweit, M., 6, 9, 11
- Kazakia, J. Y., 218
- Kerr, R. M., 296, 296 [in Ashurst *et al.* (1986)], 298 [in Ashurst *et al.* (1987)]
- Kerstein, A., 296 [in Ashurst *et al.* (1986)], 298 [in Ashurst *et al.* (1987)]
- Khaiken, S. E., 128
- Khakhar, D. V., 67, 88–94, 155, 158, 164, 165, 167–8, 169, 177, 178, 183, 185 Fig. 7.3.10 (see color plates), 187, 188, 190, 213, 227, 228–35, 238–45, 271–2, 305–6, 316–18
- Kim, J.-S., 128
- Kirstmannson, S. S., xi
- Kolmogorov, A. N., 146
- Koochesfhani, M. M., 13, Fig. 1.3.5 (see color plates)
- Krylov, V. I., 224
- Kusch, H. A., 272
- Lamb, H., 63, 139, 304
- Landford, O. E., 127, 129
- Leal, L. G., 41, 67, 85, 299, 301, 302–3, 306–8, 315, 317
- Lee, D.-I., 218
- Lee, L. J., 13
- Leonard, A., 143
- Leong, C. W., 201, 216, 219, [in Ottino *et al.* (1988)], 218
- Levich, V. G., 313
- Lichtenberg, A. J., 103, 116, 127–8, 137, 152, 217
- Lieberman, M. A., 103, 116, 127–8, 137, 152, 217
- Lightfoot, E. N., 6, 22, 316
- Lighthill, M. J., 57
- Lo, T. W., 304–5
- Lorenz, E. N., 126
- Lumley, J. L., 40, 316, 318
- Lundgren, T. S., 57, 295
- MacKay, R. S., 152–3
- MacKley, M. R., 308
- Macosko, C. S., 3, 13 [see Lee *et al.* (1980)], 13, 278, 316
- Malvern, L. E., 40
- Marble, F. E., 288
- Markovitz, H., 94–5, 339
- Marsden, J. E., 152–3
- Mason, R. G., 317
- May, R. M., 127
- McCabe, W. L., xi
- McCave, I. N., 16 [in Goldberg *et al.* (1977)]
- McKenzie, D. P., 10, 17, 217
- McMurtry, P. A., 295
- Meakin, P., 309
- Mehr, A., 221 and 271 [in Dombre *et al.* (1986)]
- Meiss, J. D., 152–3
- Melnikov, V. K., 141
- Mena, B. [see Khakhar, Chella and Ottino (1984) and Leal (1984)]

## Author index

357

- Metcalf, R. W., 295 and 295 [in McMurtry *et al.* (1986)]
- Miake-Lye, R. C., 12, 13
- Middleman, S., 16, 129, 218, 221
- Mikami, T., 317
- Milne-Thomson, L. M., 63
- Minorsky, N., 153
- Moffat, H. K., 59
- Mohr, W. D., 4, 7, 16
- Monson, P. A., 8
- Moser, J., 126, 137, 148, 152–3
- Muzzio, F. J., 313
- Nauman, E. B., 16, 317
- Noll, W., 82, 84, 94<sub>7</sub>, 339
- Nollert, M. U., 308
- O'Brien, J. J., 16 [in Goldberg *et al.* (1977)]
- Olbright, W. L., 67, 85, 305, 308
- Omohundro, S., 272
- Onsager, L., 41
- Orzag, S. A., 295
- Oseen, C. W., 95
- Ostrowsky, N. [see Meakin (1985)]
- Ottino, J. M., 3, 7, 8, 9, 13 [in Lee *et al.* (1980)], 13, 16–17, 75, 77, 81, 86, 88, 89, 90, 94, 121, 149, 155, 158, 164, 165, 167, 168, 169, 173–5, 178–80, 183, 185–6, Fig. 7.3.10 (see color plates), 188–90, 201, 213–14, 216–17, 219, 227, 228–35, 238–45, 271, 278, 280, 285, 286, 288–9, 289, 294, 305–6, 313, 315–18
- Palis, J., 153
- Pan, F., 218
- Papantoniou, D. A., 12, 13
- Patterson, A. R., 29
- Patterson, G. K., xi, 315 [in Ranz (1985)]
- Peixoto, M. M., 109
- Percival, I. C., 102, 127, 152, 153
- Perry, A. E., 35, 261–2, 266–7, 272
- Piro, O., 272
- Poincaré, H., 125, 153
- Pomphrey, N., 140
- Prandtl, L., xi, 6, 22, 40
- Rallison, J. M., 16, 67, 85, 299, 303, 306, 315
- Ralph, M. E., 251, 271
- Rangel-Nafaile, C. [see Khakhar, Chella, and Ottino (1984) and Leal (1984)]
- Ranz, W. E., 13, 13 [in Lee *et al.* (1980)], 278, 315, 316
- Reichl, L. E., 7
- Rhines, P. B., 16
- Richards, D., 102, 127, 152
- Riley, J. J., 295, 295 [in McMurtry *et al.* (1986)]
- Rising, H., 9, 121–3, 155, 158, 164, 165, 167–8, 169, 178–80, 183, 185–6, Fig. 7.3.10 (see color plates), 210, 213–14, 216, 216 [in Ottino *et al.* (1988)], 217
- Rivlin, R. S., 218
- Roberts, F. A., 138
- Roberts, J. M., 17
- Rom-Kedar, V., 143
- Rosenzweig, R. E., 47
- Roshko, A., 8
- Roughton, F. J. W., 17
- Rumscheidt, F. D., 317
- Rüssmann, H., 146, 153
- Ryu, H.-W., 218
- Saffman, P. G., 316
- Salam, F. M. A., 153
- Sander, L. M., 309
- Sax, J. E., 8, 17, Fig. 1.3.4 (see color plates)
- Saxton, R. L., 4, 7, 16
- Schlichting, H., 6
- Schowalter, W. R., 63
- Schuster, H. G., 122, 127
- Sciven, L. E., 315
- Serrin, J., 18, 20, 45–6, 52, 54, 57, 62
- Sevick, E. M., 8
- Shaw, C. D., 109, 124, 127
- Sherman, F. S., 290, 291–3, 292, 295
- Shinnar, R., 16
- Smale, S., 20, 97, 101–3, 108, 112, 115, 119, 126–9, 136
- Smith, J. C., xi
- Snyder, W. T., 272
- Sobey, I. J., 220, 249, 252–5, 271
- Sofia, S., 6
- Solomon, T. H., 216
- Soward, A. M., 221 and 271 [in Dombre *et al.* (1986)]
- Spalding, D. B., 16, 271
- Spencer, R. S., 4, 6, 129
- Stanley, H. E. [see Meakin (1985)]
- Steele, J. H. 16 [in Goldberg *et al.* (1977)]
- Stewart, W. E., 6, 22, 316
- Stone, H., 303, 307, 315
- Stuart, J. T., 167
- Swanson, P. D., 216, 216 [in Ottino *et al.* (1988)]
- Swanson, W. N., 250
- Swinney, H. L., 127
- Synge, J. L., 128, 152
- Tabor, M., [9, 11, 216, 218, in Chaiken *et al.* (1986)], 150–1 and 216 [in Berry *et al.* (1979)]
- Tadmor, Z., 16
- Tan, Q. M. [9, 11, 216, 218, in Chaiken *et al.* (1986)]

358

*Author index*

- Tanner, R. I., 67  
Taylor, G. I., 3, 4, 7, 40, 304–5, 317  
Tietjens, O. G., xi, 22, 40  
Tennekes, H., 316  
Thirring, W., 127  
Thomas, E. L., 13  
Thomas, O. M., 273  
Torza, S., 317  
Toupin, R., 19, 24, 40, 46, 59, 62–3, 94  
Townsend, A. A., 6, 277, 297, 317  
Tritton, D. J., 63  
Truesdell, C. A., 18–19, 24, 40–1, 46, 53,  
59, 62–3, 67, 94  
Tryggvason, G., 1, 3, 316  
Turcotte, D. L., 8  
Ulbrecht, J. J., xi, 315 [in Ranz (1985)]  
van de Ven, T. G. M., 316  
van Dyke, M., 272  
Varaiya, P. P., 153  
Veronis, G., 9, 16  
Virk, P. S., 318  
Vitt, E. A., 128  
von Misses, R., 21  
Voros, A., 150–1 and 216 [in Berry *et al.*  
(1979)]  
Wallerstein, G., 6  
Walters, P., 17  
Wannier, G. H., 194, 218  
Weiss, N. O., 17  
Weissberger, A. [see Roughton and Chance  
(1963)]  
Welander, P., 4, 5, 5, 9, 217  
Wendell, D. S., 4, 5  
Wickert, P. D., 13  
Wiggins, S., 143, 152  
Wiley, R. M., 4, 6, 129  
Witten, T. A., 309  
Wollan, G. N., 3  
Zweitering, Th. N., 4, 7, 317

---

## Subject index

---

Note: A subscript following a page number indicates an endnote; for example 16<sub>2</sub>, indicates the second footnote of page 16. An italic number indicates a figure or table.

- acceleration, 20, 51
- acceleration and change of frame, 51
- accessible domain, 85, 86
- action-angle variables, 132–3, 153<sub>5,6</sub>
- active interfaces, 3, 273
- active microstructures, 273, 298
  - in chaotic flows, 307
- additional body forces due to frame transformation, 52, 306
- advection, xi
- age, 279, 289
- agitation, xi
- analytical tools in the study of mixing systems, 156
- antisymmetric tensor, 51, 327
- area preserving maps, 101
- area stretch, definition, 31
- area stretch, finite areas, 36, 37–8
- astrophysics, 6
- asymptotic efficiency, 65
- asymptotic stability, 101
- average efficiency, 65
  - definitions, 65–6
    - in the linear two-dimensional flow, 70
    - in the BV flow, 180
- average mass velocity, 22
- axisymmetric extensional flow, efficiency, 68
- Baker's transformation, 6, 119, 119, 122, 129<sub>23</sub>
- barotropic fluid, 54
- basis, 320
- behavior near elliptic points, 143
- Beltrami flows, 87
- Bernoulli equation(s), 55
- bifurcations
  - diagrams, 161
  - flip or period-doubling, 162
  - in the cavity flow, 209
  - in the tendril-whorl map, 161, 164
  - saddle-node, 162
- blending, xi, 7
- blinking vortex flow (BV), 171
  - horseshoe maps, 177
  - Poincaré sections, 172
  - stability of period-1 periodic points, 176
- body couples, 46
- body forces due to frame transformation, 52
- breakup
  - and coalescence, 16<sub>5</sub>
  - experimental studies 300
  - role in mixing, 2
  - theoretical studies, 303
- bubble, separation, 268
- building blocks, 4, 154
- BV flow, 171
- canonical transformation, 133
- capillary number, 299
- capillary wave instabilities, 317<sub>28</sub>
- cat eyes flow, 254, 292
- Cauchy's equation of motion, 46
- Cauchy-Green strain tensor, 32
- Cauchy-Schwarz inequality, 65, 332
- Cauchy-Riemann conditions, 60
- Cauchy's construction, 45
- Cauchy's equation of motion, 46
- Cauchy's tetrahedron, 45
- cavity flow, 201
- cavity flow, related studies, 218<sub>30</sub>
- center, 27, 28
- centrifugal force, 52
- change of frame, 49
- change of orthonormal basis, 328
- chaotic behavior near elliptic points, 143
- chaos
  - in dynamical systems, general references, 125–7
  - in Hamiltonian systems, 152
  - possibilities in higher dimensions, 124
- characteristic times, 279–80
- characterization of the mixed state, 7
- circulation, 53

- classical means for visualization of flows, 22  
 cluster order distribution, 313  
 coagulation in chaotic flows, 312  
 coherent structures, 8, 209  
 combustion, 5, 16<sub>10</sub>, 317<sub>18</sub>  
 complex potential, 60, 90  
 composition of motions, 32  
 computation  
     numerical requirements, 208, 292–5  
     of stretching and efficiency, 64  
 computational tools in the study of mixing systems, 156  
 conjugate lines, 176, 178–9, 200  
 conservation of mass, 42  
     Lagrangian version, 42  
     Eulerian version, 42  
 constant stretch history motions, 82  
 contact forces, 43  
 continuity equation (or mass balance), 42  
 continuous dynamical systems, 97  
 continuum mechanics, role in the analysis of mixing, xi, 6  
 contravariant components, 320  
 convective-diffusion equation, 276, 280  
 copolymerization, 286–7  
 Coriolis ‘force’, 52  
 correlation function, 129<sub>27</sub>,  
 covariant components, 320  
 creeping flow, mixing in, 11, 194, 218<sub>26</sub>  
 critical capillary number, 300–2  
 curvature of lines, 37  
 curvature of surfaces, 36  
 cutting and folding, 6  
 cycles  
     homoclinic connection, 111  
     heteroclinic connection, 111  
     hyperbolic, 107  
     manifolds of, 107, 111, 114  
 cyclic variable, 133  
 cylinder, potential flow outside a, 60, 61  
 Damköhler numbers, 283, 316<sub>16</sub>  
 decay of efficiency, 66, 67, 89  
 definitions of breakup in theoretical studies, 304, 307  
 deformation analyses for droplets  
     large deformation, 304  
     small deformation, 304  
 deformation gradient, 30, 82, 98  
 definitions of chaos, summary, 124, 129<sub>27</sub>  
 degrees of freedom, 131–2  
 diffeomorphism, 18  
 diffusion limited aggregation, 309  
 diffusion, role in mixing, 2  
 direct simulation, 296  
 discrete dynamical system, 97  
 dispersion, 185, 251–3  
 dissipative systems, 98, 138, 268  
 distribution effects, 288  
 divergence of initial conditions, 116  
 drag reduction, 307, 318<sub>31,32</sub>  
 drop breakup  
     experimental studies, 300, 317<sub>26</sub>  
     theoretical studies, 303  
 droplets  
     motion, governing equations, 299  
     very viscous, 303–4  
     slender or low viscosity, 303–5  
 dual basis, 320  
 dynamical systems, 97  
 early works in mixing, 3  
 Earth, mixing in the mantle of, 8–10  
 eccentric helical annular mixer, 244  
 efficiency of mixing, 64  
 efficiency  
     in linear three-dimensional flow, 84, 86  
     in sequences of flows, 87, 89  
     in steady two-dimensional flow, 69, 70  
 eigenvalues in linear two-dimensional flow, 84  
 eigenvalues, mapping, 159  
 elliptic point, 102, 137  
 energy equation, 47  
 equivalence among various definitions of chaos, 124  
 ergodicity, 17<sub>17</sub>, 128<sub>22</sub>  
 Euler’s acceleration, 52  
 Euler’s axiom, 43  
 Euler’s formula, 20  
 Eulerian  
     viewpoint, 20, 40, 40<sub>3</sub>, 41<sub>4</sub>  
     velocity, 45  
 examples of stretching and efficiency, 68  
 exit time, 7  
 exit time distributions, 237  
 expansion work, 47  
 experimental tools in the study of mixing systems, 156  
 extra body forces, 52  
 extruders, 90  
 families of periodic points, 103  
 fast reactions, 13, 282, 284  
 Fick’s law, 276  
 first law of thermodynamics, 48  
 fixed point, 24  
 fixed points and periodic points, 99  
 flow, 14, 18  
     and motion, terminology, 17<sub>17</sub>  
     visualization, 14, 25  
 flow classification, 14, 25  
     and maps, 97  
 flows  
     extensional flow, 68  
     near walls, 260

## Subject index

361

- simple shear, 68
  - with a special form of,  $\nabla v$ , 77
  - with  $D(\nabla v)/Dt = 0$ , 77
  - with  $D(\nabla v)/Dt$  small, 79
  - with a special form of  $F$ , 82
- fluid mechanical histories, 288
- focus, 27, 28
- folding, 107, 109
  - formation of horseshoe maps in the TW map, 170–1
  - formation of horseshoe maps in the BV flow, 181
- four-roller apparatus, 41<sub>11</sub>, 317<sub>25</sub>
- furrowed channel, mixing in, 249
- fractal construction, 5
- frame indifference, 50
- Frenet triad, 37
- Galilean transformation, 51
- general structure of integrable systems, 133
- generalized shear flow, 86
- global components of mixing, 4, 15, 96
- global chaos, 153<sub>15</sub>
- gradient coagulation, 313
- gradient with respect to present configuration, 30
- gradient with respect to reference configuration, 30
- Hamel flow, 76, 77, 80, 81
- Hamiltonian, 152<sub>1</sub>
- Hamilton's equations, 130
- heat spots, 6, 277, 279
- helical annular mixer flow, 71
- helicity, 59, 269
- heteroclinic
  - connection, 111–2
  - point, 111, 112
- homeomorphism, 18
- homoclinic
  - and heteroclinic connections and points, 111
  - and heteroclinic points in Hamiltonian systems, 140
  - connection, 111–12
  - point, 111, 112, 123
- horseshoe maps, 111, 117, 119, 120, 122, 213
  - construction, 120
  - in the BV flow, 177
  - in the TW flow, 169
  - properties, 122
- hyperbolic
  - cyclic, 107, 111, 114–15, 125
  - flow, 4
  - point, 102, 137
- image analysis, 13, Figs. 1.3.4–5 (see color plates)
- immiscible fluids, 15
- impingement mixing, 13, 17<sub>16</sub>
- importance of reorientation, 87
- improper node, 27, 28
- integrable twist mapping, 133
- integrability, 14, 131
- integrability of Hamiltonian systems, 132, 134
  - intensification of vorticity, 56–7
- intensity of segregation, 8, 16<sub>2</sub>, 18<sub>3</sub>
- interactions between manifolds, 165
- interfaces
  - active, 2
  - passive, 1, 2
- intermaterial area per unit volume, 3, 278, 316<sub>1</sub>
- invariant
  - set, 128<sub>11</sub>
  - subspaces, 105
- invariants in linear three-dimensional flow, 85
- inviscid fluid, 47, 49
- irrational orbits, 143, 144, 147
- irreversibility, 180, 184
- irrotational flow, 55, 90
- isochoric flows, 19, 68, 130
- iso-concentration curves and surfaces, 3, 290
- isolation of reactants, 288
- Jacobian, 19, 121, 153<sub>4</sub>
- jets, 12
- Jordan form, 102
- journal bearing flow, 193
- Kantorovich–Galerkin method, 224
- Kelvin cat eye flow, 139, 254
- kinematical viewpoint, ix
- kinematics, xi, 40
- kinematics of deformation
  - rate of strain, 35
  - strain, 30
- Kolmogorov–Arnold–Moser theorem (KAM theorem), 146
- Kolmogorov length scales, 13, 316<sub>8</sub>
- Lagrange's theorem, 22
- Lagrangian and Eulerian viewpoints, 20, 40, 40<sub>3</sub>
- Lagrangian turbulence, x
- lamellar structures, 1, 2, 5
  - complications and illustrations, 288
  - initial conditions for concentration, 281
  - diffusion and reaction in, 274–85
  - distortion effects, 288
  - distribution effects, 289
- large deformation analyses, 304
- laser induced fluorescence, 13, Fig. 1.3.5 (see color plates), 12

- leaking from bubble, 268  
 length stretch, 31  
 linear  
     area preserving map, 101, 103  
     isochoric two-dimensional flow, 29  
     momentum, 43  
     scalar functions, 323  
     subspaces, 105  
     three-dimensional flow, 84  
     two-dimensional flow, 25, 69, 78  
     two-dimensional flow, streamlines, 27  
     vector functions, 324  
 Liapunov  
     computation in the BV flow, 180, 183  
     exponents, 111, 128<sub>21</sub>, 138  
     stability, 101  
 Liouville's theorem, 98, 128<sub>4</sub>, 135  
 Lipschitz constant, 23, 97  
 local  
     equilibrium, tractions, 45  
     flow, 298  
     processes, 4, 15  
 local stability and linearized maps, 100  
 macroscopic balance of vorticity, 57  
 macroscopic dispersion of tracer particles, 187  
 manifolds  
     of a cycle, 107, 111, 114–15  
     of hyperbolic points, 125  
 mantle of the Earth, 10  
 mass flux vector, 45  
 material description, 20  
 material  
     integrals, 36  
     volume, 42  
 matrix transformations, relationship with mixing, 7  
 maximum directions of stretching, 39  
 measurement of mixing, 7  
 mechanical energy equation, 47  
 Melnikov's method, 141  
     application examples 190, 191, 201, 255, 257  
 microflow element, 276  
 microstructure, 67, 306  
 microstructures in chaotic flows, 307  
 mixed cup average, 72  
 mixing  
     and chaos in three-dimensional and open flows, 220  
     and chaos in two-dimensional time-periodic flows, 154  
     cavity flows, 72, 201  
     mantle of the earth, 8–10  
     improvement, 91  
     in oceans, 5, 9, 16<sub>12</sub>  
     in the partitioned-pipe mixer, 221  
     in time-periodic flows, 205  
     in stars, astrophysics, 6  
     layer, Fig. 1.3.5 (see color plates)  
     mathematical definition, 117, 118, 124, 128<sub>22</sub>  
     of diffusing and reacting fluids, 15  
     of immiscible fluids, 15, 298  
     of polymers, Fig. 1.3.4 (see color plates)  
     of a single fluid, 14  
     strongly measure-theoretic, 118  
     strongly topologically, 119  
 morphological building blocks, 274  
 motion 14, 18  
 motion (velocity) around a point, 33  
 motions  
     in multicomponent media, 22  
     with constant stretch history, 82  
 multicomponent media, 22  
 Navier–Stokes equation, 47, 195, 250, 261  
 near integrable chaotic Hamiltonian systems, 149  
 Newtonian fluid, 47–8, 56, 221  
 Newton's 'second law', 152<sub>1</sub>  
 Newton's 'third law', 43  
 node, 27, 28  
 non-topological motions, 19  
 non-wandering points, 115  
 objectivity, 49  
 observability, 208  
 obstruction to dispersion, 187–9  
 oceans, mixing in, 5, 9, 16<sub>12</sub>  
 orbit, 22, 98  
 orthogonal  
     matrix, 329  
     stagnation flow, 29  
     transformation, 49  
 parabolic point, 102, 137  
 parameters and variables characterizing transport at small scales, 279  
 particle path, 22  
 partitioned-pipe mixer, 221–2  
 passive  
     interfaces, 1, 3  
     microstructures, 298  
     scalar, 292  
 pathlines, 22  
 periodic points, orbits, etc.  
     eigenvalues in the neighborhood of, 158  
     flows, 24  
     method for location of, 178–80  
     orbit, 100  
     pictorial representation, 158, 160, 161, 176  
     points, 25  
     reorientation, 83, 87

## Subject index

363

- periodicity of islands, Fig. 7.5.8 (see color plates), 232–5  
 perturbations  
     application of Melnikov's method, 190–3  
     Hamiltonian systems, 141  
 perturbed shear flow, 25, 26, 254  
 phase space, 97  
     Hamiltonian systems, 135  
     Poincaré sections and tori, 136  
 physical meaning  
     of deformation tensor, 38  
     of  $(\nabla y)^T$ , 38  
     velocity gradient, 38  
 physical picture of mixing, 1  
 Piola tensor, 41<sub>14</sub>  
 planar hyperbolic flow, 4  
 Poincaré–Birkhoff theorem, 144  
 Poincaré sections, 100, 103, 104, 105, 136,  
     137, 172, 197, 228–31, 258  
     and three-dimensional structure, 227  
     in the BV flow, 172–5  
     in the journal bearing flow, Figs. 7.4.6,  
         7.4.10 (see color plates)  
 Poincaré's recurrence, 136  
 point transformation, 18  
 point vortices in the plane, 139  
 pointed drops, 301, 304  
 Poisson's bracket, 132  
 polar decomposition theorem, 32  
 polymer mixing, Fig. 1.3.4 (see color plates), 16<sub>9</sub>  
 polymer molecules in chaotic flows, 308  
 polymer processing, 7, 13  
 polymerization, 285–7  
 porous media and chaotic flows, 308  
 possible ways to improve mixing, 91, 93  
 potential function, 59  
 present state, 19  
 principle  
     conservation of angular momentum, 46  
     conservation of energy, 48  
     conservation of linear momentum, 43  
     conservation of mass, 42  
 proper orthogonal transformation, 49  
 properties of  $e_\lambda$  and  $e_\eta$ , 66  
 qualitative picture of near-integrable  
     chaotic Hamiltonian systems, 149  
 quasi-periodic, 144  
 rate of area generation, 277  
 rates of change of material integrals, 36  
 rational orbits, 137, 143, 144  
 reattachment to wall, 268  
 reference configuration, 19, 21, 41<sub>10</sub>  
 regimes, 281, 282  
     slow reactions, 283  
 very fast reactions, 284  
 regular and chaotic flows, mixing effects,  
     186, 242–3  
 renormalization methods in Hamiltonian  
     systems, 153<sub>14</sub>  
 representation theorem, 323  
 reorientation, periodic, strong, weak, 83  
 residence time, 237–9, 240–1  
 Reynolds number, 195, 201, 249, 299  
     typical values in various processes, 9  
 Reynolds's theorem (or transport), 20, 128<sub>4</sub>  
 rotation number, 143  
 saddle, 26, 27  
 scalar vector function, 326  
 sensitivity to initial conditions and  
     Liapunov exponents, 111, 116  
 separation bubble, 263  
 sequences of flows, 87  
 shear flow  
     stretching in, 290–2, 291–7  
     time perturbation, 25  
 shear layer, 138  
 shear stress, 47  
 signatures of chaos, 111  
 simple shear flow, 4, 27, 28, 68  
 singular point, 24  
 slender body theory, 304  
 small deformation analysis, 304  
 Smoluchowski's kinetics, 315  
 spin tensor, 33  
 spiral sink, 27, 28  
 spiral source, 27, 28  
 stability  
     definitions, 101  
     of area preserving two-dimensional  
         maps, 101  
     of period-1 and period-2 periodic points  
         in the TW flow, 158  
     of periodic points in the BV flow, 158  
 stable and unstable manifolds, 105  
 standard two-dimensional cavity flow, 72  
 stars, mixing in, astrophysics, 6  
 static mixers, 129<sub>24</sub>, 223  
 statistical theory, 6  
 steady flows, 24  
 steady curvilinear flows, 82, 83  
 stirred tanks, 4  
 stirring, xi  
 stopped flow method, 17<sub>16</sub>  
 strange attractors, 129<sub>26</sub>  
 streaklines, 23, 246, 248–9  
     crossing of, 258  
     in fixed frames, 260  
     in time dependent shear flows, 26  
     with respect to moving frames, 259  
 streamfunction, 92, 130

- streamfunction and potential function, 59  
 streamlines, 23  
     in linear two-dimensional flow, 25, 27  
     in an inviscid flow, 269
- stretching  
     function, 35, 276, 280, 284  
     map, 198, Fig. 7.4.11 (see color plates), 242–3, 248  
     tensor, 33
- striation thickness, 2  
     definition, 3  
     distribution, 288  
     transport at striation scales, 273
- strong reorientation, 83
- strongly measure-theoretic mixing, 118
- strongly topologically mixing, 119
- Strouhal number, 195, 249
- structural stability, 107, 109, 110, 128<sub>15</sub>
- structured continuum, 277
- symmetry in Poincaré sections, 196–7
- symmetry of stress tensor, 46
- Taylor series expansions of flows, 260
- tendrils, 150, 151, 157, 216<sub>3</sub>
- tendril–whorl flow (TW), 155  
     formation of horseshoes, 169  
     global analysis and interactions between manifolds, 165  
     location analysis, 158  
     location and stability of period-1 and period-2 periodic points, 158
- tensor, 50
- tensors, background (see Appendix)
- test microstructures, 67
- theorems  
     Cauchy, 43  
     divergence or Gauss, 338  
     Kolmogorov–Arnold–Moser theorem (KAM theorem), 146  
     Lagrange, 22  
     Liouville, 98, 128<sub>4</sub>, 135  
     Peixoto, 109, 153<sub>7</sub>  
     Poincaré–Birkhoff, 144, 145  
     Poincaré’s recurrence, 136  
     polar decomposition, 32  
     representation, 323  
     Smale–Birkhoff, 141  
     Stokes, 339  
     transport or Reynolds, 20, 128<sub>4</sub>  
     twist, 148
- theoretic mixing, 118–19
- thermal explosions, 317<sub>19</sub>
- topological equivalence, 108, 110
- topologically mixing, 119
- tori, 136
- tracer, 3, 274
- tracking of material lines and surfaces, 292, 295
- traction, 43
- transformation, 18
- transverse homoclinic or heteroclinic intersections, 112
- trajectories, 22
- transport at striation thickness scales, 273
- transport theorem (or Reynolds), 20, 128<sub>4</sub>
- turbulent flows 6, 9, 11, 12, 13, 14, 151
- turbulent mixing layer, 8
- typical behavior of the efficiency, 66
- TW mapping, 155
- twist mapping, integrable, 133
- twist theorem, 148
- two-dimensional flows, poor efficiency, 92, 94
- two-dimensional time-periodic flows, 154
- uniformity, 187
- uniformity and irreversibility of mixing, 217<sub>23</sub>
- vector
- vectors, background (see Appendix)
- velocity, 20, 50
- velocity around a point, 33
- velocity gradient, 30
- velocity histories, 196, 196
- viscometric flows, 64
- viscous dissipation, 47, 66
- viscous liquids, 6
- visualization of manifolds, 113
- vortex, 50  
     decay, 75  
     line, 53  
     stretching by, 76  
     stretching in inviscid fluid, 57–8
- tube, 54
- vorticity  
     and angular rotation, 43, 53  
     distribution, 53  
     dynamics, 55  
     number, 67, 81  
     tensor, 33, 34
- wall displacement, 196, 198
- wandering points, 115
- warped time, 279, 316<sub>13</sub>
- weak reorientation, 67, 74
- wedge product, 143
- whorls, 151, 153<sub>16</sub>, 157, 216<sub>3</sub>
- winding number, 143