The kinematics of mixing: stretching, chaos, and transport

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Preface

The objective of this book is to present a unified treatment of the mixing of fluids from a kinematical viewpoint. The aim is to provide a conceptually clear basis from which to launch analysis and to facilitate the understanding of the numerous mixing problems encountered in nature and technology.

Presently, the study of fluid mixing has very little scientific basis; processes and phenomena are analyzed on a case-by-case basis without any attempt to discover generality. For example, the analysis of mixing and 'stirring' of contaminants and tracers in two-dimensional geophysical flows such as in oceans; the mixing in shear flows and wakes relevant to aeronautics and combustion; the mixing of fluids under the Stokes's regime generally encountered in the 'blending' of viscous liquids such as polymers; and the mixing of diffusing and reacting fluids encountered in various types of chemical reactors share little in common with each other, except possibly the nearly universal recognition among researchers that they are very difficult problems.[1]

There are, however, real similarities among the various problems and the possible benefits from an overall attack on the problem of mixing using a general viewpoint are substantial.

The point of view adopted here is that from a kinematical viewpoint fluid mixing is the efficient stretching and folding of material lines and surfaces. Such a problem corresponds to the solution of the dynamical system

$$d\mathbf{x}/dt = \mathbf{v}(\mathbf{x}, t),$$

where the right hand side is the Eulerian velocity field (a solution of the Navier–Stokes equations, for example) and the initial condition corresponds to the initial configuration of the line or surface placed or fed into the flow ($\mathbf{x}$ represents the location of the initial condition $\mathbf{x} = \mathbf{X}$). Seen in this light, the problem can be formulated by merging the kinematical foundations of fluid mechanics (Chapters 2 and 3) and the theory of dynamical systems (Chapters 5 and 6). The approach adopted here is to analyze simple protypical flows to enhance intuition and to extract conclusions of general validity. *Mixing is stretching and folding and stretching and folding is the fingerprint of chaos*. Relatively simple flows can act as prototypes of real problems and provide a yardstick of reasonable expectations for the completeness of analyses of more complex flows. Undoubtedly, I expect that such a program would facilitate the analysis of mixing problems in chemical, mechanical, and aeronautical engineering, physics, geophysics, oceanography, etc.

The plan of the book is the following: Chapter 1 is a visual summary to motivate the rest of the presentation. In Chapter 2 I have highlighted, whenever possible, the relationship between dynamical systems and kinematics as well as the usefulness of studying fluids dynamics starting with the concepts of *motion* and *flow*.[2] Mixing should be embedded in a kinematical foundation. However, I have avoided references to curvilinear co-ordinates and differential geometry in Chapters 2 and 4, even though it could have made the presentation of some topics more satisfying but the entire presentation slightly uneven and considerably more lengthy. The chapter on fluid dynamics (Chapter 3) is brief and conventional and stresses conceptual points needed in the rest of the work. The dynamical systems presentation (Chapters 5 and 6) includes a list of topics which I have found useful in mixing studies and should not be regarded as a balanced introduction to the subject. In this regard, the reader should note that most of the references to dissipative systems were avoided in spite of the rather transparent connection with fluid flows.

A few words of caution are necessary. Mixing is intimately related to flow visualization and the material presented here indicates the price one has to pay to understand the inner workings of deterministic unsteady (albeit generally periodic in this work) two-dimensional flows and three-dimensional flows in general. However, we should note that the geometrical theory used in the analysis will not carry over when $\mathbf{v}$ itself is chaotic. Though mixing is still dependent upon the kinematics, the basic theory for analysis would be considerably different. Also, even though many of the examples presented here pertain to what is sometimes called 'Lagrangian turbulence', the reader might find a disconcerting absence of references to conventional (or Eulerian) turbulence. In this regard I have decided to let the reader establish possible connections rather than present some feeble ones.

I give full citation to articles, books, and in a few cases, if an idea is unpublished, conferences. When only a last name and a date is given, particularly in the case of problems or examples, and the name does not appear in the bibliography, it serves to indicate the source of the problem

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or idea. It is important to note also that some sections of the book can be regarded as work in progress and that complete accounts most likely will follow, expanding over the short descriptions given here; a few of the problems, those at the level of small research papers are indicated with an asterisk (*). In several passages I have pointed out problems that need work. Ideally, new questions will occur to the reader.

Preface to the Second Printing

In preparing the second printing of this book, several typographical and formatting errors have been corrected. The objectives of the book expressed in the original preface remain unchanged. Owing to space constraint limitations the amount of material covered remains approximately the same. The reader interested in the connection of these ideas with turbulence will find some leads in the article 'Mixing, chaotic advection, and turbulence', *Annual Reviews of Fluid Mechanics*, **22**, 207–53 (1990); a succinct summary of extensions of many of the ideas outlined in this book is presented in 'Chaos, Symmetry, and Self-Similarity: Exploiting Order and Disorder in Mixing Processes', *Science*, **257**, 754–60 (1992). I should appreciate comments from readers pertaining to related articles in the area of fluid mixing as well as possible extensions or shortcomings of the ideas presented in this work.

Notes

1 Even the terminology is complicated. For example, in chemical engineering the terms mixing, agitation, and blending are common (Hyman, 1963; McCabe and Smith, 1956, Chap. 2, Section 9; Ulbrecht and Patterson, 1985). The terms mixing, advection, and stirring appear in geophysics; e.g., Eckart, 1948; Holloway and Kirstmannson, 1984. Inevitably, different disciplines have created their own terminology (e.g., classical reaction engineering, combustion, polymer processing, etc.).

2 Kinematics appears as an integral part of books in continuum mechanics but much less so in modern fluid mechanics. There are exceptions of course: Chapters V and VI of the work of Tietjens based on the lecture notes by Prandtl contain and unusually long description of deformation and motion around a point (Prandtl and Tietjens, 1934).

Acknowledgments

This book grew out from a probably unintelligible course given in Santa Fé, Argentina, in July 1985, followed by a short course given in Amherst, Massachusetts, also in 1985. Most of the material was condensed in eight lectures given at the California Institute of Technology in June 1986, where the bulk of the material presented here was written.

The connection between stretching and folding, and mixing and chaos, became transparent after a conversation with H. Aref, then at Brown University, during a visit to Providence, in September 1982. I would particularly like to thank him for communicating his results regarding the 'blinking vortex' prior to publication (see Secton 7.3), and also for many research discussions and his friendship during these years. I would like to thank also the many comments of P. Holmes of Cornell University, on a rather imperfect draft of the manuscript, the comments of J. M. Greene of G. A. Technologies, who provided valuable ideas regarding symmetries as well as to the many comments and discussions with S. Wiggins and A. Leonard, both at the California Institute of Technology, during my stay at Pasadena. I am also grateful to H. Brenner of the Massachusetts Institute of Technology, S. Whitaker of the University of California at Davis, W. R. Schowalter of Princeton University, C. A. Truesdell of Johns Hopkins University, W. E. Stewart of the University of Wisconsin, R. E. Rosensweig and Exxon Research and Engineering, and J. E. Marsden, of the University of California at Berkeley, for various comments and support. I am also particularly thankful to those who supplied photographs or who permitted reproductions from previous publications (G. M. Corcos of the University of California at Berkeley, R. Chevray of Columbia University, P. E. Dimotakis and L. G. Leal, both at the California Institute of Technology, R. W. Metcalfe of the University of Houston, D. P. McKenzie, of the University of Texas at Austin, A. E. Perry of the University of Melbourne, I. Sobey of Oxford University, and P. Wellander of the University of Washington).

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