

## Contents

Preface	ix
<b>1 Scalar Green functions and their perturbative solutions</b>	<b>1</b>
1.1 Quantisation of a scalar field	2
1.2 Green functions	4
1.3 The Symanzik construction for $Z[j]$	6
1.4 The series solution of the Dyson–Schwinger equations	10
1.5 The Dyson–Wick canonical series expansion	14
1.6 Coupling to an external field	16
1.7 Complex fields	20
1.8 Zero-dimensional field theory	22
1.9 Digression: The Schrödinger vacuum	23
<b>2 Connected Green functions and their one-particle irreducible components</b>	<b>26</b>
2.1 The connected Green functions and their Dyson–Schwinger equations	27
2.2 One-particle irreducibility and the effective action	33
2.3 The effective potential and its interpretation	37
<b>3 Regularisation and renormalisation</b>	<b>41</b>
3.1 Minkowski and Euclidean momentum space Feynman rules	42
3.2 Regularisation	45
3.3 Elements of perturbative renormalisation	48
3.4 The renormalisation group	53

vi	<i>Contents</i>	
<b>4</b>	<b>The scalar functional integral</b>	<b>57</b>
4.1	Functional integration	58
4.2	The Feynman series again	63
4.3	Re-ordering the Feynman series	66
4.4	A very simple model	70
4.5	Yet more rearrangements	72
4.6	Background field quantisation and the loop expansion	75
4.7	Phase-space integrals	78
<b>5</b>	<b>Series expansions and their summation</b>	<b>81</b>
5.1	The classical limit	82
5.2	The leading quantum correction	84
5.3	Series expansions	88
5.4	The formal Euclidean path integral	91
5.5	Approximate saddle-point calculations for high orders	92
5.6	Renormalisability	96
5.7	Series summation	99
5.8	Rearranging the Feynman series	102
5.9	The variationally-improved perturbation series	105
<b>6</b>	<b>Taking the path integral more seriously</b>	<b>109</b>
6.1	A first bite: finite action means zero measure	109
6.2	A second bite at the measure: non-differentiable paths	114
6.3	Saddle-points without actions	119
6.4	Minkowski theory and trouble with $i$	121
6.5	Coordinate transformations, ordering problems and rough paths	123
6.6	Ultraviolet renormalisation: the static-ultra-local model again	127
6.7	Critical behaviour, universality and dimensional regularisation	131
<b>7</b>	<b>Quantum theory on non-simply-connected configuration spaces</b>	<b>135</b>
7.1	A simple example	136
7.2	The fundamental, or first homotopy, group	140
7.3	Topological charge	143

<i>Contents</i>	vii
<b>8 Stochastic quantisation</b>	145
8.1 The zero-dimensional model	146
8.2 The scalar field	151
8.3 The stochastic field generating functional	154
<b>9 Fermions</b>	159
9.1 Fermi statistics, Green functions, and the Dyson–Schwinger equations	160
9.2 Cancellation of ultraviolet singularities: supersymmetry	167
9.3 Fermi interference and the perturbation series	169
9.4 The effective action and its renormalisation	172
9.5 Formal fermionic path integrals	173
9.6 Digression: stochastic quantisation and supersymmetry	174
9.7 Grassmannian integration: classical paths	176
<b>10 Quantum electrodynamics</b>	180
10.1 Gauge fixing	180
10.2 The electron-loop expansion	185
10.3 The infrared problem: the Bloch–Nordsieck approximation for Green functions	187
10.4 Examples of gauge invariance: $Z_1 = Z_2$ and the massless photon	192
10.5 Stochastic quantisation and gauge fixing	195
<b>11 Non-Abelian gauge theories</b>	199
11.1 The gauge principle	199
11.2 The path integral	202
11.3 Taylor–Slavnov and Becchi–Rouet–Stora identities	208
11.4 The background-field method	213
11.5 Confinement	217
<b>12 Explicit symmetry breaking and its classical limit</b>	220
12.1 Symmetry breaking and the Goldstone theorem	221
12.2 Classical Goldstone modes	223
12.3 Gauging away the Goldstone modes	227
12.4 Classical models for unification	230

<b>13 The effective potential</b>	235
13.1 The effective potential for gauge theories	235
13.2 The one-loop effective potential	236
13.3 Example: bounds on the Higgs mass	241
13.4 The Coleman–Weinberg mechanism	243
13.5 Convexity of $V$ and the failure of the loop expansion	246
13.6 Salvaging the loop expansion	248
<b>14 Field theory at non-zero temperature</b>	255
14.1 Euclidean rules at finite temperature	257
14.2 The finite temperature effective potential and phase transitions	260
14.3 Phase transitions and infrared divergences	265
14.4 Fermions and gauge fields at finite temperature	268
14.5 Coda: pure gauge fields	272
<b>15 Field theory at non-zero temperature: real-time formulation</b>	274
15.1 Real-time Feynman rules	276
15.2 Calculations in real-time	283
15.3 Canonical thermo field dynamics	284
<b>16 Instantons</b>	289
16.1 Tunnelling in quantum mechanics	289
16.2 The decay of the false vacuum	297
16.3 Periodic potentials and $\theta$ -vacua	302
16.4 Instantons and $\theta$ -vacua in non-Abelian gauge theory	305
<b>17 Composite fields and the large-<math>N</math> limit</b>	309
17.1 The $O(N)$ -invariant scalar theory at large $N$	310
17.2 Further simple models for compositeness	316
17.3 Large- $N$ adjoint representations and planarity	321
17.4 Model-making with the adjoint representation	325
<b>References</b>	330
<b>Index</b>	336