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WAVE INTERACTIONS AND FLUID FLOWS

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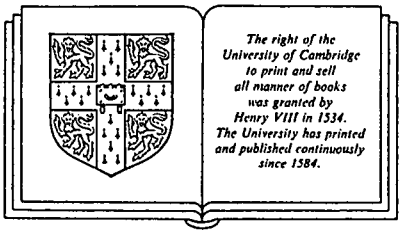

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'A wave is never found alone, but is mingled with as many other waves as there are uneven places in the object where the said wave is produced. At one and the same time there will be moving over the greatest wave of a sea innumerable other waves proceeding in different directions.'

Leonardo da Vinci, *Codice Atlantico*, c. 1500. (Translation by E. MacCurdy, *The Notebooks of Leonardo da Vinci*.)

'Since a general solution must be judged impossible from want of analysis, we must be content with the knowledge of some special cases, and that all the more, since the development of various cases seems to be the only way of bringing us at last to a more perfect knowledge.'

Leonhard Euler, *Principes généraux du mouvement des fluides*, 1755.

'Notwithstanding that...the theory is often not a little suspect among practical men, since nevertheless it rests upon the most certain principles of mechanics, its truth is in no way weakened by this disagreement, but rather one must seek the cause of the difference in the circumstances which are not properly considered in the theory.'

Leonhard Euler, *Tentamen theoriae de frictione fluidorum*, 1756/7.
(Translations by C. A. Truesdell, *Leonhardi Euleri Opera Omnia*, Ser. 2, vol. 12.)

CONTENTS

PREFACE

When, over four years ago, I began writing on nonlinear wave interactions and stability, I envisaged a work encompassing a wider variety of physical systems than those treated here. Many ideas and phenomena recur in such apparently diverse fields as rigid-body and fluid mechanics, plasma physics, optics and population dynamics. But it soon became plain that full justice could not be done to all these areas – certainly by me and perhaps by anyone.

Accordingly, I chose to restrict attention to incompressible fluid mechanics, the field that I know best; but I hope that this work will be of interest to those in other disciplines, where similar mathematical problems and analogous physical processes arise.

I owe thanks to many. Philip Drazin and Michael McIntyre showed me partial drafts of their own monographs prior to publication, so enabling me to avoid undue overlap with their work. My colleague Alan Cairns has instructed me in related matters in plasma physics, which have influenced my views. General advice and encouragement were gratefully received from Brooke Benjamin and the series Editor, George Batchelor.

Various people kindly supplied photographs and drawings and freely gave permission to use their work: all are acknowledged in the text. Other illustrations were prepared by Mr Peter Adamson and colleagues of St Andrews University Photographic Unit and by Mr Robin Gibb, University Cartographer. The bulk of the typing, from pencil manuscript of dubious legibility, was impeccably carried out by Miss Sheila Wilson, with assistance from Miss Pat Dunne.

My wife Liz, who well knows the traumas of authorship, deserves special thanks for all her understanding and tolerance; as do our children Peter and Katie, for their welcome distractions.

Many have instructed and stimulated me by their writing, lecturing and conversation: I hope that this book may do the same for others. I hope, too, that errors and serious omissions are few. But selection of material is a subjective process, and I do not expect to please everyone!

Such writing as this must often be set aside because of other commitments. But for two terms of study leave, granted me by the University of St Andrews, this book would have taken longer to complete. Things were ever so: in 1738, Colin Maclaurin wrote to James Stirling as follows –

> '...it is my misfortune to get only starts for minding those things and to be often interrupted in the midst of a pursuit. The enquiry, as you say, is rugged and laborious.'

St Andrews, September 1984