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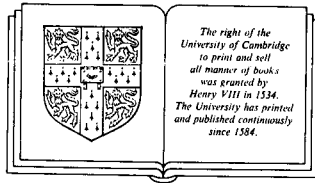
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Norman M. Martin

*Department of Philosophy
University of Texas at Austin*



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In memory of
Michael John Edward Martin (1963 –1981)

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Preface

This book is a textbook on logic. It does, however, differ in a number of respects from the greater number of its sister textbooks with which the reader may be familiar, including, very probably, the one used in the reader's first course in logic. In the first place, it is not entitled "introduction to" or any of its total or partial synonyms, such as "elements," "fundamentals," or the like. While there are several reasons for this, the most important one is that it is not an introduction, but is written primarily with an eye to students who have already been introduced to the subject, presumably in the form of a college course in modern, or symbolic, logic (there is a sense in which it is nevertheless an introduction, but that would be hair-splitting). By and large, the main (and important) purpose of such courses, and consequently of texts appropriate to them, is to teach students how to use a certain technique, modern formal logic, in the evaluation of arguments and, while doing so, to introduce some of the concepts and theory of modern logic. In consequence, most of these books rightly devote a considerable portion of their material to, for instance, the problem of translating arguments from colloquial language into the formal system or systems used (in some cases, where the course or book is intended primarily for mathematicians, common mathematical language, which is often indeed more closely related to the formal systems, is the main object of comparison, instead of "natural" language). In contradistinction to this, our main interest will be the characteristics of systems which are generally similar to those which form the basis of such an elementary course or its textbook.

Accordingly, one might say that this is a book about, rather than of, logic. Since the twenties it has been more or less customary to call a language (or part of one) used to characterize another language, a **metalanguage** and the language or system thus characterized the **object language**; and in general, to use the prefix **meta-** to characterize studies with such second-order purposes ("metaphysics", being a much older term, is of course an exception). Using this terminology, this book is thus deliberately metalogical. It has also (since

the forties) come to be customary to divide what might be called metalogic into three divisions, called, following Charles W. Morris, "pragmatics", "semantics" and "syntax" (or "syntactics"). While I have no objection to this division as indicative of one or another variety of emphasis of the particular study, I am somewhat less inclined than some to absolutize it. Presumably, syntax refers to those features which can be handled in abstraction from both the meaning of the signs and the uses to which they are put, semantics to those that include meaning considerations but abstract from use, and pragmatics to the remainder. If we allow for the fact that there are areas of interest which, depending on our emphasis, could be considered as on either side of the boundaries thus demarcated, which unlike old-fashioned European national boundaries do not have neat customs houses and striped barriers clearly marking the passage from one to another, we can, I think, accept the division for the present. In these terms, our treatment can be said to be primarily syntactical in character, although we will from time to time introduce some notions that are arguably semantic (or less frequently, even pragmatic). In most cases we shall characterize these notions so that they are capable of being considered primarily syntactical; that is, we will examine relations between two formal systems, one of which has a "natural" interpretation as a semantic system, but the characteristics actually used for that system are described in formal terms and hence it could alternatively be viewed as simply another formal system. In such cases, the real interest in the system may come from its semantic possibilities, but by virtue of the formulation, these become extraneous to the proofs, which hold even if such a system is considered to have an interpretation other than the most natural one. The main reason for this type of procedure, which may well strike the reader as peculiar (in a sense it is like playing roulette with chips without saying whether or not they will be cashed), is that it allows us to get on with the game while retaining a greater degree of neutrality with regard to the semantical and pragmatistical issues which underlie many of the disputes in the philosophy of logic. It is not our contention that these issues are not worthy of consideration, but rather that by and large, we are not interested in considering them in this book. With regard to a few of them, there will be some discussion (perhaps more out of human weakness than expository necessity) primarily in chapter 15 below. On the whole, however, we shall preserve an attitude of relative neutrality, so that the results rest on a minimum of philosophical presumptions.

For those using this as a textbook in the narrower sense, or are

otherwise relatively less experienced, it is desirable to note that the terms are introduced in, so to speak, their logically, rather than psychologically, natural order. In this connection, it is likely to be useful to read chapter 4 and perhaps chapter 5 and 6, before studying the first three chapters in full detail. Many of the concepts found in these early chapters have good "natural" examples in the systems we examine but in order to be able to introduce these concepts in the very general framework we desire, it would be very inconvenient to introduce the examples at that time.

Before starting the text it is fitting that I express my general indebtedness to the many people, colleagues and students who have helped and encouraged me during the many years that this work has been in preparation, but especially the sources and inspiration of so much of my thought: Rudolf Carnap, Stephen C. Kleene, Evert W. Beth, Alonzo Church, and Alfred Tarski. In addition I should like to thank Prof. Ignazio Angellelli for reviewing a significant part of the work; I am also indebted to four of my students, Al Carruth, M. Richard Diaz, Bernhard von Stengel, and Hardy Tichenor, for countless hours spent discussing and reviewing portions of the text. In addition, I would like to express my appreciation to Emilia R. Martin, both for typing the earlier drafts of this book and for helping edit this version of it, as well as for providing me the encouragement without which this book would never have come to be.