ANTONIO PLANS: A BIOGRAPHICAL OUTLINE

The collection of papers contained in this book is intended to be a warm homage to Antonio Plans, on the occasion of his 65 th. birthday, which at present in Spain marks the point of retirement from academic undergraduate teaching.

We have chosen the topic of Banach Spaces since it has been the center of his Mathematical interests for the last few years. The papers included have been written by friends, or students of Professor Plans, or by a few of the mathematicians who have been in touch with him for scientific reasons.

We feel sorry to restrict ourselves to just one topic, since there are many colleagues who, for sure, would have liked to write something for this event. As a matter of fact Professor Plans is widely known by his results on Knot Theory, a field he worked on at the beginning of his research career, and an active field of research today.

We now briefly sketch Professor Plans' personality and Mathematical work. The latter will necessarily be incomplete since at this very moment he is vigorously active and producing new results. For example, he is at present giving advice to four students for their doctoral dissertations.

Born in Madrid, he was the sixth of seven children. He was brought up in the cheerful atmosphere that characterizes such large families.

Soon after there came harder times; the social environment of his country was full of tension, the religious persecution started, and the growing insecurity and anxiety ended up in the civil war (1936-39). In fact, the Jesuit school he attended in Madrid was burnt down after taking the children out of their classrooms in May 1931 one month after the proclamation of the Republic. His father died in 1934 and during the war the family lost their house and their material belongings. So, when he started college at the University of Barcelona he had to find part-time jobs to contribute to the family income. Among them, he cooperated in the "Seminario Matemático" conducted by Orts Aracil, and tutored at a private Engineering school designed to complement the curriculum taught at the public university.

He graduated as a Mathematician in the University of Madrid where he had been in his last year of college. In his family there had been already

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a   long tradition of university professors; his grandfather had been a
professor at the University of Barcelona and later on he had been among
the founders of the school of Pharmacy at the University of Santiago de
Compostela.

His father, a talented man who died in his early fifties, held a
chair of   Physics at the University of Madrid; at the same time he worked
intensively in the so-called "Laboratorio Matemático" of the "Junta para
Ampliación de Estudios". This was an organization created in order to push
forward the impulse given by Rey Pastor to the Mathematical Sciences. He
was also an active member of the "Sociedad Matemática Española" and a
founder of the "Revista", a journal of those days.

As I prepared these notes, trying to find out precise data of the
Plans' family and their environment, there came to my hands a reprint of
the homage paid to his   father, José María Plans, when he died. Reading it
I realized to what extent the same words of praise could have been said of
Antonio Plans, and the great resemblance between father and son. After
glossing his scientific achievements, his contacts with Levi-Civita,
Einstein and many of the personalities of his time, and his influence in
the scientific world, Puig Adam underlines heavily his modesty, his great
concern for everybody, striving to make others shine, while he himself
remained voluntarily in the background. As a summary he says that José
María Plans was a wise man, a magister and a holy man, by his christian
virtues and his attitude towards life.

Without doubt Antonio Plans has inherited the goodness and the sharp
talent of his   father, together with the soft manners of his mother. He is
a brilliant mathematician and he is also a loyal friend. By his side
everyone feels comfortable and self confident because he always finds
one's virtues and reasons to be appreciated and   trusted. All who happen
to come in contact with him (administrative or maintenance staff members,
students etc.) are not strangers to him; he knows their names, personal
details, their needs..., and he always has an open smile   and friendly
attitude for everyone. He is also a very organized person, for whom all
the small affairs of everyday life are important and deserve
consideration.

His professional activity has always been linked with teaching. Since
1957 he has held a chair at the University of Zaragoza. Two of his most
outstanding features are his great enthusiasm for, and the attention paid

to his students. He has devoted a great deal of his life time to his undergraduate students, for whom his unlimited patience, his concern to make the hard matters easy, his clear and neat exposition in lectures, and the personal communication he established whatsoever the circumstances -overcrowded lessons, for instance- were well known.

This is even more the case with his postgraduate students. He has generously given up his time, encouraging them to go further and beyond in their work. He helps out to unthinkable limits. One of them said, in front of a tribunal of mathematicians judging his examination to become a regular professor "I owe him what I am now". This opinion could be suscribed, I think, by all of us, his students.

We give now a brief account of the ideas underlying or giving rise to his mathematical work, and the list of his publications to date. From now on all citations are referred to the latter. We do not attempt completeness due to a) the many items he has dealt with, and b) the intensive work he is developing at present in quite a number of projects.

It is very remarkable how Geometry has been his leading line; in fact he has an special ability to focus items from a geometrical point of view.

The existence of a regular basis (also called strong M-basis) in every separable Banach space is an open question which has motivated some of his research. A fundamental sequence (x_n) in a Banach space B is a <u>regular</u> <u>basis</u> if there exists a total sequence of functionals $(f_n) \subset B^*$ such that (x_n, f_n) is a biorthogonal system and for every subsequence $(n_k) \subset \mathbb{N}$, $[f_{n_k}]_\perp = [x_j]_{j \in \mathbb{N} - (n_k)}$ ([ ] stands for closed linear hull and $\perp$ for orthogonal).

Regular bases are more general than Schauder bases, and are among M-bases, which are known to exist in every separable Banach space. Papers in this line are: (1983 a), (1985), (1987 b & c), (1988) and the doctoral dissertation advised [8].

An idea of his is the so called "linear simplification" which roughly speaking consists of searching a "good" subsequence out of a given sequence. This "goodness" means amongst other things, that it must have the same closed linear span as the given one, and have some other property such as being minimal, or M-basis, or regular basis, etc. The doctoral dissertations [4] and [12] deal with it, and also (1968 a) , (1969 b), (1976), (1987 b & c).

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An auxiliary tool developed by him with one of his students, Andrés Reyes, was the lattice properties of closed linear spans of the subsequences of a given sequence, as is reflected in [8] and (1969 a), (1976), (1981), (1983 a), (1983 b). It seems appropriate here to make an special mention of Andrés Reyes, a gifted student of Plans who died prematurely in a car accident, and whose memory is very beloved to Antonio Plans and to all the members of the Department.

The linear operators acting on a Hilbert Space have been studied by A. Plans from different points of view. In relation with summability he has advised the doctoral dissertations [5] and [9] and the papers (1975 c, d & e) also deal with it. The papers (1977 a & b), (1986 a) are concerned with properties of the images of orthonormal bases. Another item studied by him are those operators A which admit a representation given by $A=\lambda U+C$, with $\lambda \in \mathbb{C}$, U and C unitary and compact operators respectively. He relates them to the hypervolume of a sequence, defined by him, and with Bari systems of vectors and rays. In this line he has advised [3] and [7] and written the papers (1964 a), (1965 b), (1966), (1967 b), (1975 a), (1979 b), (1985 b), (1988 b).

The idea of convergence has also played an important role in his research. Dealing particulary with it are (1959 c), (1964 a), (1967 c), (1973 b) and [6].

Some of his papers are concerned with General Topology, Geometric Algebra and Geometric Topology. In fact a well known result of Knot Theory is the so called Plans theorem, which refers to the homology groups of the branched cyclic coverings of a knot.

At present he is writing lecture notes on what he has called "Espacios de Apoyo". (Shuttle spaces, or Leaning spaces) which have their origin in an old result of Bertini in Projective n-dimensional Geometry. Before this he developed a theory of "unit position", as is reflected in the advising of [8] and [11], and has studied properties of the infinite-dimensional affine spaces as in [13] and (1987 a).

Finally we mention that he is a member of the "Real Academia de Ciencias" of Madrid and of Zaragoza and that he has travelled and lectured in several countries of Europe in many occasions. Presently, he is Professor Emeritus at the University of Zaragoza.

References.

Puig Adam, P.: *Jose Maria Plans. Maestro*. Las Ciencias, año II, n° 2.

PUBLICATIONS OF ANTONIO PLANS.

(1946 a) Adriano María Legendre. Matemática Elemental. VI p. 1-3.

(1946 b) Espacio de Hilbert de n dimensiones. Rev. Acad. Ciencias Madrid 40, p. 1-70.

(1947) Espacio de Hilbert. Rev. Acad. Ciencias Madrid 41, p. 197-257.

(1948) Operadores lineales en el espacio de Hilbert y su espectro. Rev. Acad. Ciencias Madrid 42, p. 309-391.

(1949) with J. Teixidor. Conferencias sobre los operadores lineales en el espacio de Hilbert. Obras completas de Gaston Julia. IV. p. 479-530.

(1952 a) Sobre la aproximación dimensional en el espacio de Kuratowski, Rev. Acad. Ciencias Madrid, XLVI, p. 303-306.

(1952 b) Algunas propiedades lineales de las matrices acotadas. Rev. Acad. Ciencias Madrid XLVI, p. 273-302.

(1952 c) Ensayo de un álgebra lineal infinita en el campo de las matrices acotadas. Coll. Math. V, p. 3-47.

(1952 d) Una forma algebraica de la dimensión de Urysohn en el espacio de Kuratowski. Rev. Acad. Ciencias Madrid VII, p. 47-50.

(1952 e) Sobre los invariantes métrico-afines de las formas cuadráticas. Gaceta Matemática IV, 7 y 8 p. 248-253.

(1953) Aportación al estudio de los grupos de homología de los recubrimientos cíclicos ramificados correspondientes a un nudo. Rev. Acad. Ciencias Madrid XLVII, p. 161-193.

(1956 a) Estudio sintético del espacio proyectivo de base no finita numerable. Publicaciones C.S.I.C, p. 1-109.

(1956 b) Primeras propiedades de las hipercuádricas en el espacio proyectivo con base no finita numerable. Rev. Mat. Hisp.-Amer. 16, p. 11-27.

(1957 a) Aportación a la homotopía de sistemas de nudos. Rev. Mat. Hisp.-Amer. 17, p. 1-14.

(1957 b) Un sistema de axiomas para el anillo de las matrices infinitas acotadas reales. Coll. Math. 9, p. 35-40.

(1957 c) Una estructura reticular del anillo de las matrices infinitas acotadas reales. Coll. Math. 9, p. 87-104.

(1958) El Tiempo. Actas de la Primera Reunión de Aproximación Filosófico-Científica, p. 133-150.

(1959 a) Propiedades angulares de los sitemas heterogonales. Rev. Acad. Ciencias Zaragoza 14, p. 5-18.

(1959 b) El Espacio. Actas de la Segunda Reunión de Aproximación Filosófico-Científica, p. 132-138.

(1959 c) Zerlegung von Folgen im Hilbertraum in Heterogonalsysteme. Arch. Math. 10, p. 304-306.

(1961 a) Resultados acerca de una generalización de la semejanza en el espacio de Hilbert. Coll. Math. 13, p. 241-258.

(1961 b) Propiedades angulares de la convergencia en el espacio de Hilbert. Rev. Mat. Hisp.-Amer. 21, p. 100-109.

(1962) Sobre los operadores lineales acotados en relación con la convergencia de variedades lineales. Coll. Math. 14, p. 269-274.

(1963 a) Los operadores acotados en relación con los sistemas asintóticamente ortogonales. Coll. Math. 14, p. 269-274.

(1963 b) Momento actual de la investigación matemática. Cuadernos de Ciencias. Universidad de Zarazoga I, 6. p. 3-11.

(1964 a) Sobre la convergencia débil en el espacio de Hilbert. Rev. Acad. Ciencias Zaragoza 19, p. 69-73.

(1964 c) Sobre una caracterización de los operadores acotados y extensiones suyas, mediante sistemas heterogonales de rayos. Actas V. R.A.M.E. p. 99-102.

(1965 a) Una caracterización de los operadores biunívocos de doble norma finita. Actas de la IV R.A.M.E. p. 119-122.

(1965 b) Sobre un determinante infinito definido mediante un operador de doble norma finita. Actas de la IV R.A.M.E. p. 123-129.

(1966) Sobre la representación de un operador lineal acotado mediante operadores isométricos y completamente continuos. Rev. Mat. Hisp.-Amer. 26, p. 202-206.

(1967 a) Sobre la continuidad uniforme angular de los operadores lineales acotados biunívocos. Actas VI R.A.M.E. p. 76-80.

(1967 b) Nuevos resultados sobre una generalización de la semejanza en el espacio de Hilbert. Actas de la VI R.A.M.E. p. 81-83.

(1967 c) Sobre el conjunto de los rayos del espacio de Hilbert. Cuadernos de Ciencias, Universidad de Zaragoza 19, p. 5-11.

(1968 a) Simplificación lineal en el espacio de Hilbert. Rev. Mat. Hisp.-Amer. 28 p. 196-199.

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(1968 b) Notas sobre la compacidad de los rayos del espacio de Hilbert.
   Actas de la IX R.A.M.E. p. 166-170.

(1969 a) Dependencias lineales en el espacio de Hilbert. Pub. Sem. Mat.
   García de Galdeano 10, p. 153-161.

(1969 b) Extensión de la dependencia lineal en el espacio de Hilbert.
   Actas de la VIII R.A.M.E. p. 90-92.

(1969 c) Espacio de Hilbert. Rev. Acad. Ciencias de Zaragoza. p. 1-40.

(1971) La topología débil del conjunto de los rayos del espacio de
   Hilbert, definida mediante hiperplanos. Rev. Acad. Ciencias Zaragoza
   26, 3 p. 525-528.

(1972) Definición de una topología en el conjunto de los rayos del
   espacio de Hilbert. Actas X R.A.M.E. p. 123-129.

(1973 a) Una base de la topología débil en el conjunto de rayos del
   espacios de Hilbert, definida mediante hiperplanos. Actas de la XI
   R.A.M.E. p. 146-154.

(1973 b) Sobre los sistemas p-heterogonales. Actas I Jornadas Mat.
   Hisp.-Lusas. p. 270.

(1975 a) Caracterización de los operadores lineales biunívocos en el
   espacio de Hilbert, que intercambian sistemas completos de rayos de
   Bari. Rev. Mat. Hisp.-Amer. 5, p. 158-166.

(1975 b) A generalization of heterogonal systems. Arch. Math. 26, 4, p.
   398-401.

(1975 c) Sobre la sumabilidad en norma en bases débiles en el espacio de
   Hilbert, mediante operadores acotados. Actas I Sem Math.
   Franco-Espagnole.

(1975 d) Transformación de sistemas heterogonales completos de rayos
   mediante operadores de Hilbert-Schmidt y nucleares. Actas I Sem. Math.
   Franco-Espagnole.

(1975 e) with Burillo, P. Operadores nucleares y sumabilidad en bases
   ortonormales. Actas I Sem Math. Franco-Espagnole.

(1976) Sistemas de vectores con núcleo nulo en el espacio de Hilbert.
   Actas de la XII R.A.M.E. p. 145-150.

(1977 a) Imagen de un sistema ortogonal completo de rayos por un operador
   lineal acotado. Coll. Math. 28, 3, p. 177-183.

(1977 b) Una caracterización de los operadores completamente continuos
   mediante sistemas ortonormales. Actas IV Jornadas Mat. Hisp.-Lusas p.
   359-363.

(1979 a) Heterogonal systems of subspaces. Proc. of the IV Int. Colloq. of Diff. Geometry, p. 225-233.

(1979 b) Sobre los sistemas ξ-heterogonales. Rev. Univ. Santander 2, p. 607-646.

(1980 a) Comportamiento de los operadores acotados en los sistemas ortogonales. Pub. Mat. Univ. Aut. Barcelona 21, p. 217.

(1980 b) Espacios de apoyo. Pub. Mat. Univ. Aut. Barcelona, 21. p. 127-128.

(1981) Propiedades inducidas por subsistemas de complemento infinito en el espacio de Hilbert. Actas VIII Jornadas Mat. Luso-Esp. p. 353-355.

(1983 a) with Reyes, A., On the geometry of sequences in Banach spaces. Arch. Math. 40 p. 452-458.

(1983 b) Another proof of a result of P. Terenzi. Proc. II Int. Conf. on Operator Algebras p. 200-202.

(1985 a) A characterization of basic sequences in Banach spaces. Symp. Aspects of Positivity in Funct. Anal. Tübingen, p. 115-116.

(1985 b) Sobre semejanza asintótica de operadores en el espacio de Hilbert. Act. Reunión de Probabilidad y Espacios de Banach, Zaragoza.

(1985 c) On the geometry of sequences in Banach spaces. Rendiconti Sem. Mat. e Fis. di Milano, LV.

(1986 a) with Martín-Peinador, E. Sistemas L de rayos y sumabilidad. Hom. Prof. Botella. Univ. Complut. Madrid. p. 203-218.

(1986 b) with García-Castellón, F. and Induráin, E. Propiedades universales de sucesiones en el espacio de Banach. Mem. Acad. Ciencias Madrid, 31, p. 1-25.

(1987 a) Un problema geométrico y afín de cuasi-complementariedad en espacios de Banach. Hom. Prof. Cid. Univ. Zaragoza, p. 173-180.

(1987 b) with García-Castellón, F. On linear simplification in Banach spaces I. Boll. U.M.I. (7) 1A. p. 87-95.

(1987 c) with García-Castellón, F. On linear simplification in Banach spaces II. Boll. U.M.I. 1A p. 367-373.

(1988 a) with Induráin, E. and Reyes, A. Notas sobre Geometría de Sucesiones en Espacios de Banach. Univ. Extremadura-Zaragoza. p. 1-262 (Book).

(1988 b) with Martín-Peinador, E. ; Induráin, E. and Rodés, A. Two geometric constants for operators acting on a separable Banach space. Rev. Mat. Univ. Complutense 1; 1,2,3, p. 25-32.

(1987) with García-Castellón and Induráin, E. Bases a través de sistemas
        minimales uniformes. Pub. Sem. Mat. G. Galdeano. Univ. Zaragoza 1-5.
(1988) with Cuadra, J.L.; García-Castellón, F.; and Induráin, E. Some
        properties of sequences with O-kernel in Banach spaces. (To appear in
        Rev. Roum. Sci.)

DOCTORAL DISSERTATIONS ADVISED.

[1] (1965) "Propiedades de una función de valores enteros, definida en el
        conjunto de las multiálgebras finitas". by Bartolomé Frontera.
[2] (1970) "Sobre el conjunto de los rayos del espacio de Hilbert". by
        Victor Onieva.
[3] (1974) "Hipervolúmenes, su límite y propiedades de permanencia en los
        operadores lineales acotados del espacio de Hilbert" by Pedro
        Burillo.
[4] (1975) "Sobre sucesiones en los espacios de Hilbert y Banach.
        Problema de la simplificación lineal topológica" by Jose Luis Cuadra.
[5] (1977) "La sumabilidad absoluta en los operadores lineales acotados
        del espacio de Hilbert" by Elena Martín-Peinador.
[6] (1978) "Convergencias en el conjunto de los subespacios del espacio de
        Hilbert" by M. Carmen de las Obras-Loscertales.
[7] (1978) "Hipervolúmenes, su convergencia y sumabilidad en los
        operadores lineales acotados del espacio de Hilbert" by Eusebio
        Corbacho.
[8] (1980) "Aspectos reticulares y geométricos de sistemas de vectores en
        espacios de Banach y de Hilbert" by Andrés Reyes.
[9] (1980) "Ideales de operadores lineales acotados y sumabilidad en el
        espacio de Hilbert" by Alvaro Rodés.
[10] (1983) "Involuciones en espacios topológicos" by Paulino Ruiz de
        Clavijo.
[11] (1985) "Sobre geometría de sucesiones en espacios de Banach.
        Posición unidad" by Esteban Induráin.
[12] (1985) "Geometría de los hiperplanos asociados a una sucesión en
        un espacio de Banach. Aplicación al problema de la simplificación
        lineal" by Felicísimo García-Castellón.
[13] (1985) "Sucesiones en la geometría afín del espacio de Banach" by
        María Jesús Chasco.