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This is the first volume of a two volume set that provides a modern account of basic Banach algebra theory including all known results on general Banach $*$ -algebras. This account emphasizes the role of $*$ -algebra structure and explores the algebraic results which underlie the theory of Banach algebras and $*$ -algebras. Both volumes contain previously unpublished results.

This first volume is an independent, self-contained reference on Banach algebra theory. Each topic is treated in the maximum interesting generality within the framework of some class of complex algebras rather than topological algebras.

In both volumes proofs are presented in complete detail at a level accessible to graduate students. In addition, the books contain a wealth of historical comments, background material, examples, particularly in noncommutative harmonic analysis, and an extensive bibliography. Together these books will become the standard reference for the general theory of $*$ -algebras.

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Frontmatter

[More information](#)

ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

- 4 W. Miller, Jr. *Symmetry and separation of variables*
- 6 H. Minc *Permanents*
- 11 W.B. Jones and W.J. Thron *Continued fractions*
- 12 N.F.G. Martin and J.W. England *Mathematical theory of entropy*
- 18 H.O. Fattorini *The Cauchy problem*
- 19 G.G. Lorentz, K. Jetter, and S.D. Riemenschneider *Birkhoff interpolation*
- 21 W.T. Tutte *Graph theory*
- 22 J.R. Bastida *Field extensions and Galois theory*
- 23 J.R. Cannon *The one-dimensional heat equation*
- 25 A. Salomaa *Computation and automata*
- 26 N. White (ed) *Theory of matroids*
- 27 N.H. Bingham, C.M. Goldie, and J.L. Teugels *Regular variation*
- 28 P.P. Petrushev and V.A. Popov *Rational approximation of real functions*
- 29 N. White (ed) *Combinatorial geometries*
- 30 M. Pohst and H. Zassenhaus *Algorithmic algebraic number theory*
- 31 J. Aczel and J. Dhombres *Functional equations containing several variables*
- 32 M. Kuczma, B. Chozewski, and R. Ger *Iterative functional equations*
- 33 R.V. Ambartsumian *Factorization calculus and geometric probability*
- 34 G. Gripenberg, S.-O. Londen, and O. Staffans *Volterra integral and functional equations*
- 35 C. Gasper and M. Rahman *Basic hypergeometric series*
- 36 E. Torgersen *Comparison of statistical experiments*
- 37 A. Neumaier *Interval methods for systems of equations*
- 38 N. Korneichuk *Exact constants in approximation theory*
- 39 R.A. Brualdi and H.J. Ryser *Combinatorial matrix theory*
- 40 N. White (ed) *Matroid applications*
- 41 S. Sakai *Operator algebras in dynamical systems*
- 42 W. Hodges *Model theory*
- 43 H. Stahl and V. Totik *General orthogonal polynomials*
- 44 R. Schneider *Convex bodies*
- 45 G. Da Prato and J. Zabczyk *Stochastic equations in infinite dimensions*
- 46 A. Björner, M. Las Vergnas, B. Sturmfels, N. White and G. Ziegler *Oriented matroids*
- 47 G.A. Edgar *Stopping times and directed processes*
- 48 C. Sims *Computation with finitely presented groups*

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Frontmatter

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*Banach Algebras and
The General Theory of *-Algebras
Volume I: Algebras and Banach Algebras*

THEODORE W. PALMER

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Frontmatter

[More information](#)

PREFACE

This volume provides a gentle introduction to most of the main areas of research on general Banach algebras. It also serves the more specific purpose of providing the background for Volume II which will deal more intensively with *-algebras (*i.e.* algebras with fixed involutions, normally denoted by $*$). The focus is on the algebraic, and sometimes the geometric, underpinnings of the analytic theory. The subject is rich with aesthetic appeal, and many topics are pursued just as far as I found them attractive. References are given to more thorough expositions when they are available or to original sources. I have tried to make the book readable for beginning graduate students. Towards this end, I sometimes include a bit of undergraduate level material when it may not have been absorbed by such readers. There are also generous comments and historical remarks. They are all intended to serve a pedagogic purpose. I have tried to document the original source of most ideas, but sometimes I have failed. I apologize to those thus slighted. The knowledgeable reader will also find numerous previously unpublished results and technical improvements.

Readers should note the Symbol Index at the end of the volume. I have chosen notation carefully and used it consistently throughout the work. For instance, \mathcal{A} always represents an algebra and \mathcal{A} with a subscript always represents a subset of that algebra. Each entry in the bibliography displays the numbers of the sections in this volume to which it is related. A few of these entries, primarily those recording recent papers, are not actually referred to in the text but have been included to record the names of current research workers. When it is convenient to state several parallel cases in a single definition or result, the various options are enclosed in angle brackets $\langle \rangle$ and separated by an ordinary slash $/$. Internal references are given by the familiar device of tripartite numbers separated by periods where the \langle first / second / third \rangle number refers to the \langle chapter / section / subsection or statement \rangle . Additional notation and conventions are introduced at the beginning of Chapter 1.

In 1970 I began writing a book on *-algebras. Repeatedly I discovered that existing references did not cover the background material in sufficient detail or from the viewpoint I needed. Thus this first volume began life as

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Frontmatter

[More information](#)

vi

Preface

a series of appendices. Chapter 2 on the spectrum and spectral algebras is a direct descendent of the first of these appendices which attained the status of a complete independent exposition. When the appendices became as long as the main text, I realized that they needed to come first and could easily be expanded to provide a relatively complete introduction to Banach algebra theory. I learned this theory from the ground-breaking book of Charles E. Rickart [1960] and those familiar with his book will see the strong influence his organization of the subject still has on my own.

By 1978 I had written a relatively complete manuscript. Unfortunately it was never quite finished for publication, and I devoted the decade of the 1980's mainly to administrative work. During that whole period, I tried to keep current with work on general Banach algebras, and I continually revised sections and incorporated really striking new results which obviously belonged. However, there was no time to complete the manuscript for publication. A daunting pile of typed pages was the result in those pre-computer days. Finally it became nearly impossible to trace down and change all the cross references when new material was added.

In December 1987 Robert S. Doran (Texas Christian University) asked me how my book was progressing and whether I had a publisher. Wholly involved with dean's work at the time, I replied that I did not see how I would ever finish it without a coauthor. Within a few days he expressed willingness to revise the book as a coauthor and I immediately accepted. Bob quickly arranged a contract with Cambridge University Press, and I began to withdraw from further administrative commitments. In December 1988 Bob sent me a preprint of Thomas J. Ransford's beautiful proof of Barry Johnson's uniqueness of norm theorem. Within a few hours I had used Ransford's method to give a new proof of the fundamental theorem of spectral semi-norms (Theorem 2.3.6). Since I had known for years that an easy early proof of this result was a key to a smooth exposition of many of my ideas, I decided then and there to leave the dean's office and work to complete this book. With Bob's help it seemed a task of two or three years at most. Unfortunately, during the 21 months it took me to free myself of all administrative commitments, Bob was drawn more and more into administration himself. In March 1991 he had to drop his role as coauthor. This book would have greatly benefitted had he been able to continue. Besides securing a contract for publication, Bob worked with his colleague David Addis to develop $\text{T}_{\text{E}}\text{X}$ macros for the book and arranged to have his wife Shirley Doran prepare a preliminary $\text{T}_{\text{E}}\text{X}$ version of my old manuscript. Without all their contributions, I could not have $\text{T}_{\text{E}}\text{X}$ ed the whole book. Bob also made numerous suggestions on style. Many readers will thank him for convincing me to give up the use of the Fraktur alphabet. (In my heart, I still believe that \mathfrak{A} is a typical Banach algebra.)

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Frontmatter

[More information](#)

Preface

vii

Many other people have helped with completion of this book. All the words and mistakes are my own, but most of the commas were contributed by my wife Laramie and by Kenneth A. Ross, both of whom have proof-read essentially the whole work. Richard M. Koch has repeatedly come to my rescue when some computer or \TeX nicality defeated me. Laramie has helped with the book from the beginning; Ken and Dick were also essential. John Duncan (University of Arkansas), Robert B. Burckel (Kansas State University) and Michael J. Meyer (Georgia State University) read most of the manuscript, sometimes in earlier versions, and made valuable suggestions. Barry E. Johnson (University of Newcastle) and H. Garth Dales (Leeds University) helped on more limited portions. Numerous colleagues have provided preprints or valuable information. Beginning in 1970 several generations of graduate students at the University of Oregon have seen preliminary versions of the book. They have pointed out obscurities or even errors and in the later stages have contributed to the proofreading efforts. I cannot list them all but here are a few: Abdullah H. Al Moajil, Robert Bekes, Michael Boardman, Sean Bradley, Jon M. Clauss, David Collett, Dan Hendrick, Thomas W. Judson, Michael Leen, Chung Lin, Jorge M. López, Michael Ottinger, William L. Paschke, Paul L. Patterson III, John Phillips, James Rowell, Richard C. Vrem and Sheng L. Wu. To all those mentioned above by name or category, I extend heart-felt appreciation.

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Frontmatter

[More information](#)

With deep filial respect, this volume is dedicated to:

Ernest Jesse Palmer

(April 8, 1875 to February 25, 1962)

and

Elizabeth McDougall Palmer

(March 14, 1902 to April 25, 1972).

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Banach Algebras, Volume I - Theodore W. Palmer

Frontmatter

[More information](#)

CONTENTS

Preface	v
1 Introduction to Normed Algebras; Examples	1
1.1 Norms and Semi-norms on Algebras	2
1.2 Double Centralizers and Extensions	24
1.3 Sums, Products and Limits	37
1.4 Arens Multiplication	46
1.5 Algebras of Functions	64
1.6 Matrix Algebras	69
1.7 Operator Algebras	78
1.8 Group Algebras on T	111
1.9 Group Algebras	128
1.10 Tensor Products	158
2 The Spectrum	187
2.1 Definition of the Spectrum	192
2.2 Spectral Semi-norms	208
2.3 The Jacobson Radical and the Fundamental Theorem	224
2.4 Spectral Algebras	233
2.5 Spectral Subalgebras and Topological Divisors of Zero	245
2.6 Numerical Range in Banach Algebras	260
2.7 The Spectrum in Finite-Dimensional Algebras	272
2.8 Spectral Theory of Operators.	286
2.9 Topological Algebras	297
3 Commutative Algebras and Functional Calculus	301
3.1 Gelfand Theory	303
3.2 Šilov Boundary, Hulls and Kernels	319
3.3 Functional Calculus	341
3.4 Examples and Applications of Functional Calculus	356
3.5 Multivariable Functional Calculus	378
3.6 Commutative Group Algebras	417

Cambridge University Press

0521366372 - Banach Algebras and The General Theory of *-Algebras: Algebras and
Banach Algebras, Volume I - Theodore W. Palmer

Frontmatter

[More information](#)

x	Contents	
4	Ideals, Representations and Radicals	437
4.1	Ideals and Representations	440
4.2	Representations and Norms	456
4.3	The Jacobson Radical	474
4.4	The Baer Radical	481
4.5	The Brown-McCoy or Strong Radical	490
4.6	Subdirect Products	495
4.7	Categorical Theory of Radicals	498
4.8	History of Radicals and Examples	503
5	Approximate Identities and Factorization	519
5.1	Approximate Identities and Examples	519
5.2	General Factorization Theorems	534
5.3	Countable Factorization Theorems	543
6	Automatic Continuity	553
6.1	Automatic Continuity of Homomorphisms into \mathcal{A}	555
6.2	Automatic Continuity of Homomorphisms from \mathcal{A}	571
6.3	Jordan Homomorphisms	579
6.4	Derivations	591
6.5	Jordan Derivations	614
7	Structure Spaces	619
7.1	The Hull-Kernel Topology	619
7.2	Completely Regular Algebras	632
7.3	Primary Ideals and Spectral Synthesis	639
7.4	Strongly Harmonic Algebras	645
8	Algebras with Minimal Ideals	655
8.1	Finite-Dimensional Algebras	657
8.2	Minimal Ideals and the Socle	667
8.3	Algebras of Operators with Minimal Ideals	672
8.4	Modular Annihilator Algebras	678
8.5	Fredholm Theory	688
8.6	Algebras with Countable Spectrum for Elements	695
8.7	Classes of Algebras with Large Socle	699
8.8	Examples	711
	Bibliography	715
	Index	783
	Symbol Index	791

Cambridge University Press

0521366372 - Banach Algebras and The General Theory of $*$ -Algebras: Algebras and
Banach Algebras, Volume I - Theodore W. Palmer

Frontmatter

[More information](#)

Tentative Contents of Volume II

9 $*$ -Algebras

- 9.1 $*$ -Algebras
- 9.2 $*$ -Representations
- 9.3 Density Theorems
- 9.4 Topologically Cyclic $*$ -Representations
- 9.5 C^* -semi-norms
- 9.6 Topologically Irreducible $*$ -Representations
- 9.7 The Reducing Ideal and $*$ -Representation Topology
- 9.8 Hermitian and Symmetric $*$ -Algebras
- 9.9 Linear Maps Between $*$ -Algebras

10 Special $*$ -Algebras

- 10.1 G^* -algebras and BG^* -algebras
- 10.2 T^* -algebras, S^* -algebras, and Sq^* -algebras
- 10.3 U^* -algebras
- 10.4 Hermitian S^* -algebras and γS^* -algebras
- 10.5 $*$ -Regularity
- 10.6 $*$ -Algebras with Minimal Ideals

11 Banach $*$ -Algebras

- 11.1 Fundamental Results on $*$ -Representations
- 11.2 Unitary Structure of Banach $*$ -Algebras
- 11.3 Continuity of Positive Linear Functionals
- 11.4 Ideal Structure
- 11.5 Hermitian Banach $*$ -algebras
- 11.6 Hilbert Algebras and Tomita Algebras
- 11.7 Banach $*$ -Algebras with Minimal Ideals

12 Locally Compact Groups and their $*$ -Algebras

- 12.1 Group $*$ -Algebras
- 12.2 Classes of Locally Compact Groups
- 12.3 Structure Theory
- 12.4 Representation Theory
- 12.5 Ideal Theory

Cambridge University Press

0521366372 - Banach Algebras and The General Theory of *-Algebras: Algebras and
Banach Algebras, Volume I - Theodore W. Palmer

Frontmatter

[More information](#)**13 Cohomology of Banach algebras**

- 13.1 Basic Results and the Long Exact Sequence of Cohomology
- 13.2 Reduction to the Unital Case
- 13.3 Amenable Banach Algebras
- 13.4 Amenable Groups and Cohomology of Group Algebras
- 13.5 Cohomology and Extensions
- 13.6 Cohomology and Fixed Points
- 13.7 Diverse Examples, Applications and Extensions

14 K-Theory of Banach Algebras

- 14.1 K-Theory of \mathbb{Z}_2 -graded Banach Algebras
- 14.2 The Cyclic Exact Sequence
- 14.3 $K_0(\mathcal{A})$ and $K_1(\mathcal{A})$ in the Ungraded Case
- 14.4 Applications of K-Theory