Addressing the need for new models for the analysis of social network data, Philippa Pattison presents a unified approach to the algebraic analysis of both complete and local networks. The rationale for an algebraic approach to describing structure in social networks is outlined, and algebras representing different types of networks are introduced. Procedures for comparing algebraic representations are described, and a method of analysing the representations into simpler components is introduced. This analytic method, factorisation, yields an efficient analysis of both complete and local social networks.

The first two chapters describe the algebraic representations of the types of networks, and the third chapter covers the ways in which representations of different networks can be compared. A general procedure for analysing the algebraic representations is then introduced, and a number of applications of the approach are presented in the final chapters.

The book should be of interest to all researchers interested in using social network methods.

Structural analysis in the social sciences 7

Algebraic models for social networks

Structural analysis in the social sciences

Mark Granovetter, editor

Other books in the series:

Ronald L. Breiger, ed., *Social Mobility and Social Structure*
John L. Campbell, J. Rogers Hollingsworth and Leon N. Lindberg, eds., *Governance of the American Economy*
David Knoke, *Political Networks: The Structural Perspective*
Kyriakos Kontopoulos, *The Logics of Social Structure*
Mark S. Mizruchi and Michael Schwartz, eds., *Intercorporate Relations: The Structural Analysis of Business*
Barry Wellman and S. D. Berkowitz, eds., *Social Structures: A Network Approach*

The series *Structural Analysis in the Social Sciences* presents approaches that explain social behavior and institutions by reference to *relations* among such concrete social entities as persons and organisations. This contrasts with at least four other popular strategies: (a) reductionist attempts to explain by a focus on individuals alone; (b) explanations stressing the causal primacy of such abstract concepts as ideas, values, mental harmonies and cognitive maps (thus, "structuralism" on the Continent should be distinguished from structural analysis in the present sense); (c) technological and material determinism; (d) explanations using "variables" as the main analytic concepts (as in the "structural equation" models that dominated much of the sociology of the 1970s), where structure is that connecting variables rather than actual social entities.

The social network approach is an important example of the strategy of structural analysis; the series also draws on social science theory and research that is not framed explicitly in network terms, but stresses the importance of relations rather than the atomisation of reductionism or the determinism of ideas, technology or material conditions. Though the structural perspective has become extremely popular and influential in all the social sciences, it does not have a coherent identity, and no series yet pulls together such work under a single rubric. By bringing the achievements of structurally oriented scholars to a wider public, the *Structural Analysis* series hopes to encourage the use of this very fruitful approach.

Mark Granovetter

Algebraic models for social networks

Philippa Pattison
University of Melbourne

CAMBRIDGE
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, Victoria 3166, Australia

First published 1993

Library of Congress Cataloging-in-Publication Data

Pattison, Philippa.

Algebraic models for social networks / Philippa Pattison.

p. cm. – (Structural analysis in the social sciences: 7)

Includes bibliographical references and index.

ISBN 0-521-36568-6

1. Social networks – Mathematical models. I. Title. II. Series.
HM131.P3435 1993
301'.01'51 – dc20

92-34916
CIP

A catalog record for this book is available from the British Library.

ISBN 0-521-36568-6 hardback

Transferred to digital printing 2003

To my parents

Contents

Figures and tables

Figures

Tables

Preface

A class of models for analysing social network data are described in this work. The models are offered in response to two related needs arising from current developments in social science research. Firstly, data on social networks are being gathered much more commonly, a fact that is reflected by the inclusion in 1985 of a set of network questions in the General Social Survey (Burt, 1984). As a result, there is a growing need for a variety of models that will enable the analysis of network data in a number of different forms. Secondly, the role of social networks in the development of social and psychological theory is increasingly prominent and calls for the development of data models attuned to a variety of theoretical claims about the nature of that role.

Arguments for the importance of social networks can be found in both the psychological and sociological domains. Social psychologists have documented their dissatisfaction with the "differential" view of social behaviour embodied in many psychological theories (e.g., Cantor & Kihlstrom, 1981; Fiske & Taylor, 1990; Harre & Secord, 1972; Magnusson & Endler, 1977; Moscovici, 1972) and have argued for an analysis of social behaviour that is more sensitive to the "meaningful" context in which it occurs. One aspect of that context is the network of social relations in which the behaviour in question is embedded, a contextual feature to which empirical studies of some kinds of behaviour have already given explicit recognition (e.g., Henderson, Byrne & Duncan-Jones, 1981).

On the sociological side, the case for the importance of social networks was initiated much earlier, and those studies that demonstrated the salience of social and personal networks have become classics (e.g., Barnes, 1954; Bott, 1957; Coleman, Katz & Menzel, 1957). Indeed, a considerable amount of attention has been devoted to the problems of obtaining information about social networks and representing it in some explicit form (e.g., Fischer, 1982; Harary, Norman & Cartwright, 1965; Henderson et al., 1981; Laumann, 1973; White, Boorman & Breiger, 1976). Moreover, in addition to their role in making social context explicit, social networks have played a significant part in the "aggregation" problem, a role that Granovetter (1973), in particular, has made clear. The aggregation problem is the process of inferring the global, structural implications of local, personal interactions (White, 1970). Granovetter has demonstrated that the problem is not straightforward and has shown in several instances how an understanding of the local social network assists the task of inferring macro level social behaviour (Granovetter, 1973; also, Skog, 1986).

The models for which an analysis is developed in this book have therefore been chosen to be sensitive to these two main themes for the role of social networks in social theory: as an operational form of some aspects of social context and as a vehicle for the aggregation of local interactions into global social effects. The claim is not made that the models selected are unique in filling this role, although it will be argued that their properties are closely aligned with a number of theoretical mechanisms proposed for them.

The starting point for the models is the characterisation of social networks in terms of blockmodels by White et al. (1976) and the subsequent construction of semigroup models for role structure (Boorman & White, 1976; also, Lorrain & White, 1971). In presenting the construct of a blockmodel as a representation for positions and roles from multiple network data, White et al. argued that it was necessary to develop a view of concrete social structure that did not depend on the traditional a priori categories or individual attributes in the sociologists' battery but rather on the networks of relations among individuals. They claimed that blockmodels provide a means for representing and ordering the diversity of concrete social structures, and they showed how the semigroup of a blockmodel provides a representation of its relational structure at a more abstract, algebraic level.

Later, Winship and Mandel (1983) and Mandel (1983) extended the blockmodel framework to include a representation for what they termed "local" roles. In so doing, they decoupled the notion of local role from the global role structure approach of Boorman and White, thus pointing the way to an algebraic characterisation of role in incomplete or ego-centred networks.

In this book, I have attempted to develop an integrated method of analysis for these and some related algebraic characterisations of role structure in social networks. I argue that the algebraic description of structure is natural from the perspective of social theory and extremely useful from the perspective of data analysis. In particular, it allows for a general means of analysing network representations into simple components, a property that greatly enhances the descriptive power of the representations. A major theme of the work is that the provision of such a means of analysis is a necessary precursor to adequate practical evaluation of the representations. Moreover, an eventual by-product of this form of analysis should be a catalogue of commonly occurring structural forms and the conditions under which they occur and, hence, a more systematic development of projects initiated by Lorrain and by Boorman and White in their accounts of simple structural models.

The first two chapters describe the algebraic representations adopted for complete and local networks, respectively. The question of which networks possess identical algebraic representations is addressed in chapter 3, together with the more general question of how to compare the algebraic representations of different networks. In chapter 4, a general procedure for analysing the algebraic representations of complete and local networks is described. The task of relating this analysis to aspects of network structure is taken up in chapter 5, where a number of illustrative applications of the overall analytic scheme are also presented. Chapter 6 contains an application of the scheme to complete and local networks measured over time, while chapter 7 presents the algebraic representations that can be constructed for valued network data. Finally, chapter 8 discusses the contribution of the analysis to some important issues for network analysis, including the description of positions and roles, structural models for networks and the comparison of network structures.

The work has benefited from the assistance of many people. Warren Bartlett lent a great deal of encouragement and support in the early stages of the work, and Harrison White and Ronald Breiger have given help in many different ways over a number of years. Many of the ideas developed here have their source in earlier work by Harrison White and François Lorrain and also by John Boyd; the work also owes much to many insightful commentaries by Ron Breiger. I am grateful, too, to Stanley Wasserman for his helpful remarks on two drafts; and I am especially indebted to my family – Ian, Matt and Alexander, my parents and my parents-in-law – for their help and patience.