

I

*Thoughts on the true estimation of
living forces and assessment of the
demonstrations that Leibniz and other
scholars of mechanics have made use of in
this controversial subject, together with
some prefatory considerations pertaining to
the force of bodies in general*

INTRODUCTION

In 1686, in a short article published in the *Acta Eruditorum* and titled “A Brief Demonstration of Memorable Errors of Descartes and Others Concerning a Natural Law,” Leibniz claimed to demonstrate that one of Descartes’s fundamental laws of motion was false.¹ Specifically, Descartes held that, due to God’s immutability, the ‘quantity of motion’ in the world must be conserved, where the quantity of motion was to be represented as the product of the size and the speed of matter in motion. Translated into contemporary terms and modified somewhat, this quantity is called ‘momentum’ and is represented by mv .² Moreover, Descartes’s law of the conservation of the quantity of motion formed an integral part of his broader philosophical position, not only because it followed immediately, on his view, from the necessity of God’s immutable nature, but also because it had to be consistent with Descartes’s distinctive and rather restrictive account of the nature of matter, namely as consisting solely in extension, including its modes, such as size, shape, place, and changes therein such as motion. For whatever quantity God conserves in the world must be a quantity that matter actually has, and since size and velocity are modes of extension, Descartes’s account of matter goes hand in hand with his conservation law. As a consequence, however, if Leibniz’s objection to Descartes’s conservation law is correct, then it does not concern an inessential detail of Descartes’s position, but rather goes to the heart of his natural philosophy and entails that significant features of that account must be rejected.

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Leibniz's explicit argument, which is presented in his *Discourse on Metaphysics* (§ 17), proceeds by way of a consideration of the following three principles. (1) A body that falls from a certain height acquires, through its fall, the same force that is necessary to elevate it to that same height (excluding external interference, such as friction with the air, etc.). This principle is sometimes viewed as a more specific instance of the metaphysical-sounding law that the whole effect must be equal to the total cause. (2) The same quantity of motion, which Descartes also referred to as motive force, is required to raise a body with one unit of mass to a height of four units of length (call this case A) as is required to raise a body with four units of mass to a height of one unit of length (call this case B). This principle follows from Descartes's law of the conservation of the quantity of motion, since it entails that the quantity of motion of the bodies in cases A and B are equal; for 1 times 4 ($m_a v_a$) is the same as 4 times 1 ($m_b v_b$). This principle may seem to be intuitive, since it would not appear to make any difference to the force involved whether one raises one body one unit of length four times in succession or rather raise four such bodies one unit of length each. (3) Galileo proved experimentally that the velocity a body acquires in free fall is proportional to the square of the distance fallen. The problem, Leibniz argues, is that these principles are inconsistent. While the first and second principles entail that the quantities of motion in cases A and B are equal, the first and third principles entail that the quantity of motion in case B would have to be greater than the quantity of motion in case A. Specifically, according to Galileo's law, the velocity acquired if the ball is released in case A is twice the velocity acquired by the ball if released in case B, but since the body's mass in case B is four times greater than the body's mass in case A, the quantity of motion in case B will be twice as great as that in case A. According to Leibniz's argument, therefore, the quantity of motion is not conserved in cases of bodies in free fall and Descartes's conservation law is false.³

While Leibniz thus concluded that the 'quantity of motion' (mv) is not conserved in such cases, he did not for that reason conclude that no quantity at all is conserved in the world. Instead, he suggested that something he called 'motive force' is conserved, though this quantity is represented as the product of the mass and the square of the velocity (i.e., mv^2) and was also referred to as living force. In contemporary terms, this quantity is partially captured by our concept of kinetic energy ($=1/2 mv^2$). Moreover, throughout the 1680s and '90s, Leibniz developed a novel and comprehensive natural philosophy that was designed, at least in part, to support this conservation law. Thus, in "A New System of the Nature and Communication of Substances, and of the Union of the Soul and Body," published in 1695, he articulated the fundamental features of the nature of substance as an active force that could serve

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not only as a metaphysical principle of unity (in contrast to Descartes's infinitely divisible extension), but also as the seat of such a living force, whereas in the first part of his "Specimen Dynamicum," likewise published in 1695, he advanced a dynamical account of bodies, positing primitive and derivative active and passive forces as part of his analysis, thereby allowing him to arrive at what he called "a true estimation of forces."⁴

The controversy that ensued, the so-called *vis viva* debate, was considerable, unsurprisingly so, given that two comprehensive natural philosophies were at stake. Many of the most important figures working in natural philosophy and mathematics at the time weighed in on the issue, with the sides lining up, roughly, according to nationality; the French usually agreed with Descartes, whereas the Germans, Dutch, and Swiss mostly followed Leibniz. The English Newtonians either remained neutral on the issue (e.g., by rejecting the idea that any quantity must be conserved) or sided with the Cartesians. (It may be recalled that Leibniz and Newton did not enjoy particularly friendly relations after their public controversies, e.g., about the discovery of the calculus.) Moreover, in spite of the apparent simplicity of the cases that were invoked on each side, no explicit consensus emerged for several decades about how best to resolve the dispute. In fact, while many scholars have claimed that d'Alembert articulated the definitive solution in his *Traité de Dynamique* in 1743 (according to which the problem arises due to an ambiguity in the way in which terms, such as 'motive force,' are used), others have claimed more recently that the dispute did not rest on a simple confusion or ambiguity that could be clarified in short order and that the dispute ended not so much with a clear resolution as with an eventual lack of interest.⁵

Viewed against this broader philosophical background, the central point of Kant's *Thoughts on the True Estimation of Living Forces* can be summarized as a sustained attempt at resolving this debate. However, the situation is hardly this simple and straightforward, even if one abstracts from the difficulty involved in finding a coherent and satisfying resolution of the conflicting principles and arguments. For the circumstances surrounding the writing and publication of this work were rather complicated. Kant started working on the *True Estimation* as a twenty-one-year-old student around 1744 and completed most of it in 1746, at which time he submitted it to the university censor in Königsberg, who approved it for publication. However, publication, financed in part by Kant and in part by a close relative, was delayed for three years, until 1749, which allowed Kant to insert further material (a dedication as well as further argument and commentary in §§ 107–113 and §§ 151–156) in 1747. In 1746, however, Kant's father died after a lengthy illness, leaving him, as the eldest son, with the task of dealing with the family's estate

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and the care of his siblings, as well as an even less favorable financial situation than he had faced previously. In these circumstances, Kant left the university in August of 1748, without obtaining a degree, to become a private tutor to a series of families in the vicinity of Königsberg. That Kant did not receive a degree was due, at least in part, to the fact that he had not written a suitable master's thesis; the *True Estimation* was written in German, not in Latin, as would have been required at the time.

These complications suggest several questions about the *True Estimation*. For one: Why did Kant write and then publish the *True Estimation* at all, especially when it came at considerable personal expense and at a time when he found himself in an unfavorable financial situation? What did he hope to achieve with an abstract academic treatise on a topic in natural philosophy that did not contribute to advancing his career at the university? It was clearly an expression of intellectual independence and grand ambitions. It might also be interpreted as an act of rebellion against his teachers who may have failed to appreciate his talents.⁶ It is significant that Martin Knutzen, one of Kant's teachers, recommended other students over Kant, and that Kant may well have been criticizing Knutzen's position in the first part of the book (though he is not explicitly mentioned by name).

For another: Who was Kant's intended audience for the book? The fact that it was written in German (rather than Latin or French) would have excluded the widest possible European audience. Yet his remarks in the preface suggest that he hoped for a broad readership. Kant's own actions provide an oblique indication of his intentions. After the book was published, he sent a copy to a former fellow student, Ferdinand Mühlmann, requesting that it be reviewed, and another copy to Leonhard Euler, the famous mathematician at the Royal Academy of Sciences in Berlin at the time. If Euler had thought well of the work, he could have improved Kant's prospects considerably. Whatever Kant's intended purpose and audience, however, the book received a favorable review by Mühlmann in the *Frankfurtische Gelehrte Zeitung* in 1749, a satirical review by Lessing in the *Neuestes aus dem Reich des Witzes* in 1751, and an anonymous critical review in the *Nova Acta Eruditorum* in 1752.

What is Kant's main argument in the book? The *True Estimation* is divided into a preface and three chapters. In the preface, Kant makes the case that his thoughts should be taken seriously, despite the fact that he was not a well-known author and he was addressing a highly contentious issue. Specifically, he expressed his intention of contradicting and criticizing a number of the leading intellectuals of the day (I–II), claimed that prejudice, though an ineradicable element of the human condition, will not deter him from subjecting his thoughts to the impartial judgment of others (III–VII), and addressed the concern that he might appear to be overly confident or, for that matter, impolite in the

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statement of his views (VIII–X). He ended with a brief assessment of the current state of the controversy regarding the proper measure of force (XI–XIII), where he concluded, with a somewhat misguided pre-science, that the controversy “will be settled shortly, or it will never cease” (1:16).

In Chapter One, “Of the force of bodies in general,” Kant considers the proper notion of force in general and distinguishes two different kinds of motions that are fundamental, in his view, to resolving the *vis viva* debate. He begins by arguing that the force that is essential to bodies should be characterized, with Aristotle and Leibniz, as an active, and not, with Wolff, as a moving force, even though one can explain how an active force is responsible for motion (§§ 1–4). In addition to the fact that one can avoid the circularity of invoking *moving* force to explain *motion*, Kant thinks that it allows one to solve the mind–body problem (§§ 5–7), to explain the relations between substances and the world they constitute through their causal connections (§§ 8–11), and to clarify two objections that have been raised against a certain understanding of how forces act on each other (§§ 12–14). Along the way, Kant offers suggestions about how causality is prior to spatiality (§ 9) and how the three-dimensionality of space derives from the inverse proportionality of the square of the distances (§ 10). He concludes the first chapter by distinguishing between a ‘free’ motion, which can conserve itself in the body to which it has been communicated and which can therefore continue to infinity if no impediment opposes it, and those motions that require constant external stimulation and thus which disappear immediately if their external forces cease to sustain them (§§ 16–19). He has projectiles in mind as examples of the former, and what was recognized as ‘dead force’ as an instance of the latter. This distinction turns out to be crucial, because it will allow him to advance his “main purpose of improving on the Leibnizian measure of force” (1:28). Specifically, he wants to argue that both kinds of motion are real, with the former requiring living force, represented by the Leibnizian concept of ‘living force’, mv^2 (our ‘energy’), and the latter, by contrast, needing dead force, which is represented by the Cartesian measure, mv (our ‘momentum’).

Chapter Two, “Examination of the theorems of the Leibnizian party concerning living forces,” by far the longest chapter, is an extensive and detailed critique of Leibniz’s position and of the various arguments he and his followers had advanced in its favor. Kant’s main reason for accepting the Cartesian measure over the Leibnizian one – with important qualifications to which Kant returns in Chapter Three – is that the Cartesian conception of force is measurable in bodily motions over time and in space, whereas Leibnizian force pertains only to an incipient stage prior to motion that for that reason cannot be measured experimentally (§§ 20–28). The bulk of the chapter is devoted to an analysis

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of the range of relevant mechanical cases. In §§ 30–36, Kant argues that Leibniz cannot use the case of free fall to support his position, since he fails to take into account Descartes's condition that the time during which the fall occurs is relevant to a proper analysis, and both Herrmann's and Lichtscheid's responses on Leibniz's behalf are shown to be inadequate. In §§ 37–57 Kant argues for three separate claims: (1) the various accounts of the collisions of elastic bodies that are equal in their mass and velocity offered by Herrmann, Bernoulli, and Chastelet are unsatisfactory, since rather than supporting the conservation of 'living forces,' such cases actually prove the Cartesian estimation; (2) his objections are not to 'living force' per se, but rather to the more limited point that 'living forces' could be measured mathematically; (3) he shows that the complications arising in cases of unequal bodies make no relevant difference to the case in favor of 'living forces.' In §§ 58–70, Kant then reacts critically to Leibniz's account of cases of inelastic collisions. In §§ 71–113 Kant proceeds to analyze a range of more complicated cases: compound motions (§§ 71–78), oblique and circular motions (§§ 79–85), as well as further cases discussed by Leibniz (§§ 92–102), Wolff (§§ 103–106), Musschenbroek (§§ 107–108), and Jurin, Chastelet, and Richter (§§ 109–113). The second chapter concludes with miscellaneous remarks about previously discussed issues.

In Chapter Three, "Presenting a new estimation of living forces, as the true measure of force in nature," Kant presents his own resolution of the conflict between the Cartesian and the Leibnizian measures of force. Central to his account is the distinction between free and unfree motions he had introduced in Chapter One, and a corresponding distinction between natural and mathematical bodies, for this allows him to assert that even though the Cartesian estimation of force is mathematically correct for certain kinds of bodies in motion, the Leibnizian estimation is also correct, albeit not mathematically, for certain other kinds of bodies in motion. In §§ 114–137 Kant lays out the basic elements of his account, including an explanation of how vivification occurs through the infinitely many steps from dead to living force (§§ 122–123), a statement of his own new law, without conditions (§124), a clarification of the contingent status of living forces (§129), and the discovery of "a completely unknown dynamical law" which, he alleges, is even confirmed by experience (§§ 132–133). In §§ 138–150 Kant then clarifies how his account applies to a range of cases, many of which he had analyzed to a different end in Chapter Two: how living force relates to external resistance (§ 138), gravity (§§ 139–140), soft bodies (§ 141), varying masses (§§ 142–145), fluids (§§ 146–147), and elastic bodies (§§ 148–149). In §§ 151–156, one of the later additions, Kant inserts a critical discussion of Musschenbroek's 'mechanical' proof of living forces. In §§ 157–163, Kant concludes this chapter, and thus his first published work, with a

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‘proof’ of his theory and a discussion of objections that he anticipates being leveled against his position in light of remarks made by various Cartesians.

Kant’s *Thoughts on the True Estimation of Living Forces* cannot be viewed as having achieved what he had hoped for it. It did not solve the *vis viva* debate, and many of his most distinctive claims have been rejected.⁷ However, the work does provide a substantive view of Kant’s earliest philosophical thought, which is interesting in its own right as well as extremely useful for understanding Kant’s later, more revolutionary Critical period.

This translation, which is the first one to be published in English, is based on a reprint of the original published edition, though the version printed in the Academy edition has been consulted and several emendations suggested therein have been indicated in footnotes. For ease of use, references to the Academy edition are placed in the margins to the text. Factual notes are indebted in numerous places to Kurd Lasswitz’s “*Sachliche Erläuterung*” [Factual Explanations] in the Academy edition.⁸

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