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Axioms of cooperative decision making



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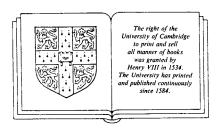
Two-sided matching: A study in game-theoretic modeling and analysis



Axioms of cooperative decision making

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Foreword

Amartya Sen

Two of the main developments in social analysis, broadly defined, since World War II have been the emergence of social choice theory and the blossoming of the theory of games. Each development was characterized by a major contribution that set the stage for further explorations. John von Neumann and Oskar Morgenstern established the fruitfulness of using the theory of games as a tool for the analysis of economic behavior, social cooperation, and strategic analysis. Kenneth Arrow's analysis of social choice and individual values established a new approach to the understanding and investigation of social aggregation, political mechanisms, and normative economic judgments. Both made extensive and elegant use of mathematical reasoning and, inter alia, demonstrated the fruitfulness of formal, axiomatic methods for substantive analysis of problems of practical importance. Each of these lines of investigation led to the development of a vast – often rather technical – literature.

Given the close connections between the subject matters of the two fields of investigation, it would have been natural to expect that there would be a lot of interchange between the two and a good deal of crossfertilization. There have, in fact, been a number of attempts at drawing lessons from one field for the other, but it is fair to say that much of social choice theory and the bulk of game theory tend to proceed on their own, without even a side glance at the other field. There are, of course, exceptions.³

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¹ J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior* (Princeton University Press, 1944).

² K. J. Arrow, Social Choice and Individual Values (New York: Wiley, 1951; second edition, 1963).

³ The literature on the manipulability of social choice mechanisms has to be inescapably concerned with game-theoretic issues in social-choice-theoretic contexts. Beginning with early contributions from such authors as Robin Farquharson and Michael Dummett, the literature received a tremendous boost when the powerful Gibbard-Satterthwaite theorem



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Hervé Moulin has, in fact, been at the forefront of both social choice theory and game theory, and this monograph is a remarkably illuminating account of the uses that can be made of formal social choice theory and formal game theory in understanding and analyzing the nature and the process of cooperative decision making. Most problems of social interaction involve both conflict and congruence of interests. The elements of congruence make it important to have cooperation, but there are many forms that cooperation can take, and the interests of the different parties may diverge in the choice between different cooperative arrangements. Moulin discusses these problems with remarkable lucidity as well as reach.

Moulin begins with a clear exposition of a particular approach, namely, "welfarism," which has been extensively used in social choice theory, in which social welfare is taken to be a function of – and only of – individual utilities. The utilities can be combined in different ways (e.g., by adding, as under utilitarianism, or by concentrating on the utility of the worst-off individual, as under egalitarian rules), but they share the same concentration on individual utilities only. Part I of the book provides an exploration of this approach and its connections with the problem of bargaining, leading to the selection of a particular outcome from a class of alternative cooperative outcomes that are differentially beneficial to different parties. Moulin's approach here – as elsewhere in the book – is axiomatic, and he makes excellent use of clarifying the principles involved in the different axioms that can be employed to systematize, analyze, and critically evaluate alternative solutions.

In Part II Moulin shifts his focus from social choice theory to game theory, dealing specifically with problems of cooperation. The insights to be gained from game-theoretic analysis are fully used, supplementing what had been learned from the use of axiomatic social choice theory.

In Parts III and IV Moulin discusses, respectively, the problem of public decision making and that of voting and social choice. The former includes important problems of public economics (such as the choice of taxation systems, the regulation of monopolies, problems of decentralization), and the latter is concerned with different political mechanisms, involving various types of voting systems and other ways of dealing

Footnote 3 (cont.)

on the impossibility of nonmanipulable social choice mechanisms (more generally, game forms) was established. Results very similar to those of Gibbard were also established around that time by Pattanaik, with some variations. The subsequent literature has been extensive, with many interesting impossibility and positive possibility results. See particularly Hervé Moulin, *The Strategy of Social Choice* (Amsterdam: North Holland, 1983), and Bezalel Peleg, *Game Theoretic Analysis of Voting in Committees* (Cambridge: Cambridge University Press, 1984).



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with aggregation of partially conflicting preferences of different parties. Throughout these analyses, Moulin makes efficient and elegant use of lessons learned from social choice theory as well as game theory. The overall result is to leave the reader immensely better informed about the issues involved in cooperative decision making and how they might be handled, with an eye both to substantive relevance and to the possibility of using formal reasoning in analyzing problems that may initially look too complex and messy.

There are two features of the book I would particularly like to draw attention to in this preliminary statement. First, a good deal of social choice theory has been concerned primarily with presenting impossibility results, showing what cannot be done rather than what can be. Even though the subject began with the stunning impossibility theorem presented by Kenneth Arrow, to concentrate primarily on deriving impossibility results is quite the wrong way of seeing social choice theory. One object of noting an impossibility is to question the initial choice of axioms and to suggest what variations in the axiomatic structure should be considered. This leads to positive possibilities – what can be done and how. Moulin's approach is constructive throughout, and in this respect the monograph must be seen as an important contribution to creative formal thinking, using social choice theory as well as game theory.

Second, in fields that involve a good deal of formal reasoning, the extent of communication is severely limited by the difficulty of mastering exactly what is going on in the process of deduction that may look remote and unduly abstract. Moulin's method of dealing with this problem is to enrich the analysis with an enormous number of examples, which bring out precisely what is involved. He also supplements his presentations by a number of useful and well-chosen exercises, making sure that the reader has got to the bottom of the problem. As a pedagogic exercise this strategy of presentation is quite excellent, and the book should be praised for that reason as well.

It should be clear to the reader that I am an unashamed admirer of what Moulin has done. His efforts will undoubtedly be well rewarded if the readers pay attention to this work and benefit from it. I do not doubt at all that this will happen in abundance.



Acknowledgments

This book emerged from several graduate classes on collective decisions, social choice, and cooperative games, taught mostly at the Université de Paris Dauphine, Princeton University, and the Virginia Polytechnic Institute. I wish to thank all my students for their patience and criticisms. Three of my colleagues provided invaluable advice at various stages of this manuscript, for which I am most grateful. They are John Roemer, William Thomson, and Peyton Young.

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