

Cambridge University Press

978-0-521-35947-4 - Nonstandard Analysis and its Applications

Edited by Nigel Cutland

Table of Contents

[More information](#)

## CONTENTS

<i>Preface</i>	ix
<i>Contributors</i>	xiii
<b>AN INVITATION TO NONSTANDARD ANALYSIS</b>	<b>1</b>
<b>Tom Lindstrøm</b>	
INTRODUCTION	1
CHAPTER I. A SET OF HYPERREALS	4
1 Construction of ${}^*\mathbb{R}$	4
2 Internal sets and functions	10
3 Infinitesimal calculus	17
CHAPTER II. SUPERSTRUCTURES AND LOEB MEASURES	22
1 Superstructures	22
2 Loeb measures	27
3 Brownian motion	38
CHAPTER III. SATURATION AND TOPOLOGY	48
1 Beyond $\aleph_1$ -saturation	48
2 General topology	52
3 Completions, compactifications, and nonstandard hulls	59
CHAPTER IV. THE TRANSFER PRINCIPLE	68
1 The languages $L(V(S))$ and $L^*(V(S))$	68
2 Łoś' theorem and the transfer principle	73
3 Axiomatic nonstandard analysis	81
APPENDIX ULTRAFILTERS	84
NOTES	90
REFERENCES	99

Cambridge University Press

978-0-521-35947-4 - Nonstandard Analysis and its Applications

Edited by Nigel Cutland

Table of Contents

[More information](#)

vi

CONTENTS

<b>INFINITESIMALS IN PROBABILITY THEORY</b>	<b>106</b>
<b>H. Jerome Keisler</b>	
1 The hyperfinite time line	107
2 Universal and homogeneous probability spaces	109
3 Stochastic processes	112
4 Products of Loeb spaces	114
5 Liftings of stochastic processes	117
6 Adapted probability spaces	122
7 Adapted distributions	125
8 Universal and homogeneous adapted spaces	131
9 Applications to stochastic analysis	136
References	139
 <b>INFINITESIMALS IN FUNCTIONAL ANALYSIS</b>	 <b>140</b>
<b>C. Ward Henson</b>	
1 Topological vector spaces	142
2 Operators	159
3 Uniform equivalence	164
4 Indiscernibles	166
5 Isomorphic nonstandard hulls	175
References	180
 <b>APPLICATIONS OF NONSTANDARD ANALYSIS IN MATHEMATICAL PHYSICS</b>	 <b>182</b>
<b>Sergio Albeverio</b>	
1 Introduction	182
2 Singular interactions in Schrödinger operators: a case study	184
3 Nonstandard theory applied to closed bilinear forms on Hilbert spaces	190
4 Dirichlet forms as standard parts of hyperfinite Dirichlet forms, and applications to quantum mechanics	194
5 Hyperfinite energy forms, diffusions and quantum mechanics on fractals	200
6 Polymer measures	203
7 Quantum fields: hyperfinite models and connections with polymer measures	205

Cambridge University Press

978-0-521-35947-4 - Nonstandard Analysis and its Applications

Edited by Nigel Cutland

Table of Contents

[More information](#)

<b>CONTENTS</b>	<b>vii</b>
8 Other topics and conclusions	211
References	213
<b>A LATTICE FORMULATION OF REAL AND VECTOR VALUED INTEGRALS</b>	<b>221</b>
<b>Peter A. Loeb</b>	
1 Scalar functions and measures	222
2 Internal functionals on continuous functions	229
3 Vector functions and measures	232
References	235
<b>AN APPLICATION OF NONSTANDARD METHODS TO COMPUTATIONAL GROUP THEORY</b>	<b>237</b>
<b>B. Benninghofen and M.M. Richter</b>	
Introduction	237
1 Group theoretic preliminaries	237
2 The growth function and automata	240
3 The nonstandard hull of $FN(a,b)$	243
4 Non-regularity results	248
References	257
<b>SYNTACTICAL METHODS IN INFINITESIMAL ANALYSIS</b>	<b>258</b>
<b>Francine Diener and Keith D. Stroyan</b>	
1 Introduction	258
2 IST: an alternate axiomatization of set theory	259
3 Bounded internal set theory	263
4 The bounded formulas (T), (I), (S).	266
5 Quantifier reduction rules	271
6 Generalized transfer and idealization	275
7 Permanence principles	278
References	280
<b>SOME ASYMPTOTIC RESULTS IN ORDINARY DIFFERENTIAL EQUATIONS</b>	<b>282</b>
<b>Francine and Marc Diener</b>	
1 Properties of the trajectories	283
2 $\epsilon$ -shadows expansions of implicit functions	287
3 Existence of an expansion	291
4 Conclusion: application to the problem of streams	293
References	295

<b>SUPERINFINITESIMALS AND INDUCTIVE LIMITS</b>	<b>298</b>
<b>Keith D. Stroyan</b>	
1 Monads	298
2 Product monads	300
3 Limit spaces and topologies	303
4 Limit vector spaces	305
5 Limit infinitesimals	308
6 Examples	315
7 Counterexamples	317
References	320
<b>THE NON-LINEAR BOLTZMANN EQUATION FAR FROM EQUILIBRIUM</b>	<b>321</b>
<b>Leif Arkeryd</b>	
1 Introduction	321
2 The Boltzmann equation	322
3 Loeb solutions to the Boltzmann equation	325
4 The Maxwellian limit	335
References	340
<b>INDEX</b>	<b>341</b>