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The right of the University of Cambridge to print and sell all manner of books was granted by Henry VIII in 1534. The University has printed and published continuously since 1584.

Cambridge University Press

Cambridge
New York   New Rochelle
Melbourne   Sydney

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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521257541

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First published 1985
First paperback edition (with corrections) 1988
Re-issued in this digitally printed version 2008

A catalogue record for this publication is available from the British Library

Library of Congress Catalogue Card Number: 84-14309

ISBN 978-0-521-25754-1 hardback
ISBN 978-0-521-35872-9 paperback

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Introduction

The subject of design theory has grown out of several branches of mathematics, and has been increasingly influenced in recent years by developments in other areas. Its statistical origins are still evident in some of its standard terminology (thus 'v' for the number of points in a structure comes from 'varieties'). Today it has very fruitful connections with group theory, graph theory, coding theory and geometry; these ties have been two-way, by and large.

We have attempted in this book to lay the groundwork for an understanding of designs, with advanced undergraduate or postgraduate students in mind. Our aim is to prepare the reader to use designs in other fields or to enter the active field of designs themselves. Finite projective and affine geometries are central to design theory, and are introduced early in the book. Since classical geometry is a very large field, the student with a background in this subject will be at an advantage, but we have tried to present a treatment sufficiently self-contained to answer the needs of a reader with a reasonable knowledge of linear algebra. The subject of symmetric designs is also introduced early, and its important aspects (the Bruck–Ryser–Chowla Theorem, Singer groups and difference sets, Hadamard 2-designs, etc.) are developed. The first four chapters, covering basic definitions, geometry and symmetric designs, are designed to be part of any course based on the book.

The other four chapters can be studied more or less independently of one another. Chapter 5 covers resolvable and affine designs; Chapter 6 introduces 2-designs other than those met already; Chapter 7 deals with 1-designs and an introduction to generalised quadrangles; finally Chapter 8 studies the large Mathieu designs and groups.

Certain families of designs (e.g., projective planes or Hadamard designs) occur so often in design theory that they merit special treatment. Sometimes even particular single designs play such an important role that we have dealt with them in considerable detail (e.g. the 2-design for (11, 5, 2), the biplanes on 16 points, the projective plane of order 4). The little Mathieu designs (in Chapter 4) and the large Mathieu designs (in Chapter 8) take up a large amount of the book. We have chosen also to emphasise the connections with

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group theory, where some of the most beautiful and important theorems of design theory can be demonstrated. But we have tried to isolate the less elementary group theory so that a reader without the necessary background (which is not very great, in any case) can cope with it satisfactorily. Another important connection is between graph theory and designs: we include a self-contained introduction to the elementary theory of strongly regular graphs in Chapter 3.

The authors have found that an undergraduate one-term course can be based on the first four chapters. A slightly more advanced (or longer) course can add one of the later chapters. At a substantially higher level much, or even all, of the last four chapters can be covered.

Many different notational and terminological usages have grown up in the theory of designs. We have attempted to lay out a consistent scheme for both of these and, excepting in comments, we have ignored the notations etc. not used in the book. 'Structures' are very primitive objects, perhaps having the relationship to designs that groupoids do to groups. We feel that repeated blocks are not of central interest in the theory (and are only of marginal interest to the 'users' of design theory) and, except in certain very special circumstances, the book reflects this attitude. Some terminologies from the past might best be altered (e.g. 'group-divisible designs', which have nothing to do with groups, and can merely be called 'divisible designs'). We have almost universally used upper case Latin letters for points and lower case for blocks (an exception is in geometries); structures, designs and sets in general, are normally upper case script letters, groups are upper case Latin but their elements either lower case Latin or lower case Greek, while graphs are usually indicated by upper case Greek letters (an exception being the Hussain graphs of Chapter 3).

We have indicated with a '*' those exercises whose solutions are more difficult than the ordinary, or perhaps just considerably longer. There are also a few 'Problems': this means that we do not know the answer but that there might be some profit in investigating the situation.

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