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978-0-521-35868-2 - Elements of Functional Analysis, Second Edition

I. J. Maddox

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ELEMENTS OF FUNCTIONAL ANALYSIS

I. J. MADDUX

*Professor of Pure Mathematics
in the Queen's University of Belfast*

Second edition



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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521358682

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First published 1970
Second edition 1988
Reprinted 1993, 1994
Re-issued 2011

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-35350-2 Hardback
ISBN 978-0-521-35868-2 Paperback

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This book is dedicated to the memory of

EDITH MADDOX (1877–1958)

May it serve as an epitaph

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PREFACE TO THE SECOND EDITION

Since the first edition of 1970 the subject of functional analysis has continued its natural development and expansion. In 1973 the famous basis problem of Banach was solved by Enflo, who proved that there is a separable Banach space that has no basis. Unfortunately, many recent advances in the subject do not fall within the elementary sphere, so that there has been little opportunity to include very recent work in this text, whose paramount purpose is to maintain its introductory character.

In the preparation of the second edition I have been encouraged by the interest shown in the first edition, whose aim was to provide a really introductory, though non-trivial, course for those who had completed basic courses on real and complex variable theory.

The material in this new edition should be even more accessible to the beginner, with the clarification of many topics, the insertion of more detail in explanations and proofs and the addition of further worked examples.

There has been a complete revision of the work on convex sets, metric and topological linear spaces, reflexivity and weak convergence. Also, there is new material on the Wiener algebra of absolutely convergent Fourier series, and on weak topologies, including a proof of Alaoglu's theorem.

Elementary applications of functional analysis to differential and integral equations appear in the new final chapter, including a treatment of the famous Sturm–Liouville problem. It is my hope that these topics will be of particular value and interest to applied mathematicians, physicists and engineers.

The first edition contained a fairly extensive final chapter on matrix transformations in sequence spaces. Although this topic is still close to my heart I consider now that it is rather too specialized to receive lengthy treatment in an introductory text. Nevertheless the present edition contains a short account of matrix transformations, including the very interesting theorem of Schur, which carries the corollary that weak and strong sequential convergence are equivalent in the space of absolutely convergent series.

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Finally, it is a pleasure for me to acknowledge the work of Mrs Patricia Gilliland, who cheerfully typed my manuscript with great speed and accuracy.

I. J. MADDOX

Queen's University of Belfast, 1987

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PREFACE TO THE FIRST EDITION

There are several excellent books which deal with the subject of functional analysis. Few can be regarded as really elementary or introductory. As a beautiful theory in its own right and for its richness in applications, functional analysis in some shape or form is now taught to second and third year mathematics undergraduates at several British universities. My experience in teaching such students has indicated that they need quite a gentle introduction – largely due to two things: that their analytical abilities are not sufficiently developed and that they are unused to ‘abstract’ reasoning. In my view, the field of elementary functional analysis is the ideal place in which to learn some abstract structural mathematics and to develop analytical technique.

It is my hope that this book may provide a really introductory, though non-trivial, course on functional analysis for undergraduates who have completed basic courses on real and complex variable theory. Although primarily addressed to students of mathematics it is expected that the approach is basic enough to enable students of physics and engineering to get something of the flavour of the subject.

Of the several excellent books mentioned above, the master work of Banach: *Théorie des opérations linéaires* (1932) must stand first. Every serious student of analysis should regard his education incomplete until he has read something of this remarkable germinal book.

There is one feature of the present work which we should perhaps mention. Much of the theory is illustrated by examples involving sequence spaces rather than integration spaces. This is partly because most results for sequence spaces will fairly readily generalize to integration spaces, but mainly because the student to whom this book is addressed is unlikely to be sufficiently familiar with integrals of the depth of Lebesgue to enable him to really appreciate examples involving them. However, it has been thought advisable to prove the completeness of the important L_p spaces, referring to works on integration for the relevant theorems on interchange of limit and integral.

Chapter 1 of the book is absolutely fundamental, though extremely elementary. Some may wish to omit it and proceed to the next chapter on metric and topological spaces. In my view it would be best to make certain

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of the material in chapter 1 before attempting the rest of the text. There are over 300 exercises in the book, many of which are quite routine, though just a few, which appear at the end of the last chapter, are quite difficult. It is recommended that most of the exercises should be attempted – to learn mathematics one must do it.

The final chapter of the book concerns an area of mathematics which is of special interest to me. Those students who wish to begin graduate work in this field may find it a useful introduction. Readers who are not so inclined may, nevertheless, see functional analysis at work in a fairly concrete situation.

Debts of gratitude are several. At the undergraduate level my interest in analysis was stimulated by Professor D. C. Russell. As a research student I was greatly influenced by my supervisor, Dr B. Kuttner. A number of my colleagues at the University of Lancaster have made helpful comments on the book, during many conversations. I am especially indebted to P. L. Walker for his careful scrutiny of the typescript and for numerous valuable suggestions. Useful assistance was also rendered by J. W. Roles and C. G. Lascarides.

The manuscript was expertly typed by Mrs Sylvia Brennan and Miss June Unsworth, and I gratefully acknowledge their help.

I. J. MADDOX

University of Lancaster, 1969