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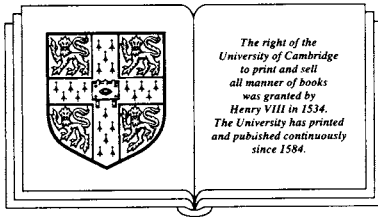
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# Cohen-Macaulay Modules over Cohen-Macaulay Rings

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Dedicated to Professor Hideyuki Matsumura on his sixtieth birthday.

## Contents

Chapter 1.	Preliminaries	1
Chapter 2.	AR sequences and irreducible morphisms	8
Chapter 3.	Isolated singularities	16
Chapter 4.	Auslander categories	25
Chapter 5.	AR quivers	35
Chapter 6.	The Brauer-Thrall theorem	44
Chapter 7.	Matrix factorizations	54
Chapter 8.	Simple singularities	60
Chapter 9.	One-dimensional CM rings of finite representation type	69
Chapter 10.	McKay graphs	85
Chapter 11.	Two-dimensional CM rings of finite representation type	98
Chapter 12.	Knörrer's periodicity	106
Chapter 13.	Grothendieck groups	118
Chapter 14.	CM modules on quadrics	124
Chapter 15.	Graded CM modules on graded CM rings	135
Chapter 16.	CM modules on toric singularities	143
Chapter 17.	Homogeneous CM rings of finite representation type	161
	Addenda	168
	References	170

## Preface

This is a widely revised version of lectures I gave at Tokyo Metropolitan University in 1987, originally written in Japanese [67].

Throughout the book our attention is directed to the point – how we can classify CM modules over a given CM ring, or classify CM rings which have essentially a finite number of CM modules? Being analogous to lattices over orders, this question seems to have arisen very naturally. The first approach to CM modules in this direction was, perhaps, done by Herzog [35], and it turned out that there are, in themselves, two basic aspects of this problem. The first is an algebraic or representation-theoretic side, in which Auslander and Reiten made remarkable progress by the powerful use of AR sequences. The second is a geometric side, more precisely, the spectra of CM local rings having only a finite number of CM modules should be well-behaved singularities. Artin, Verdier, Knörrer, Buchweitz, Eisenbud, Greuel and many others are concerned with this direction.

In this book I have tried to give a systematic treatment of the subject, as self-contained as possible, but there is no intention to make this book an encyclopedia of CM modules. Therefore, even in the case that more general treatments for definitions or proofs exist, I have preferred to give direct expositions, which, I am afraid, the experienced reader might feel unwise of me.

I conclude this preface with acknowledgements and thanks to all who supported the preparation of the book. Especially I would like to express my great gratitude to Prof. Maurice Auslander who gave me valuable comments on the first draft. I am also indebted to Eduardo Marcos, Akira Ooishi and Kazuhiko Kurano who read the manuscript with great care and attention. Finally I wish to thank Mr. David Tranah, an editor of Cambridge U.P., for his advice in correcting my English.

*January, 1990*

*Yuji Yoshino*