

## Contents

### Preface xvii

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Evolution of engineering materials	1
1.2	Fiber composite materials	1
1.3	Why composites?	10
1.3.1	Economic aspect	10
1.3.2	Technological aspect	12
1.4	Trends and opportunities	17
1.5	Microstructure–performance relationships	19
1.5.1	Versatility in performance	20
1.5.2	Tailoring of performance	22
1.5.3	Intelligent composites	24
1.6	Concluding remarks	26
<b>2</b>	<b>Thermoelastic behavior of laminated composites</b>	<b>29</b>
2.1	Introduction	29
2.2	Elastic behavior of a composite lamina	30
2.2.1	Elastic constants	30
2.2.2	Constitutive relations	33
2.3	Elastic behavior of a composite laminate	39
2.3.1	Classical composite lamination theory	39
2.3.2	Geometrical arrangements of laminae	44
2.4	Thick laminates	46
2.4.1	Three-dimensional constitutive relations of a composite lamina	46
2.4.2	Constitutive relations of thick laminated composites	48
2.5	Thermal and hygroscopic behavior	54
2.5.1	Basic equations	55
2.5.1.1	Constitutive relations	55
2.5.1.2	Thermal and moisture diffusion equations	61
2.5.2	Hygroscopic behavior	62
2.5.2.1	Moisture concentration functions	62
2.5.2.2	Hygroscopic stress field	65

2.5.3	Transient interlaminar thermal stresses	67
2.5.3.1	Transient temperature field	67
2.5.3.2	Thermal stress field	68
2.5.4	Transient in-plane thermal stresses	73
2.5.4.1	Transient temperature field	74
2.5.4.2	Thermal stress field	77
<b>3</b>	<b>Strength of continuous-fiber composites</b>	<b>80</b>
3.1	Introduction	80
3.2	Rule-of-mixtures	81
3.3	Stress concentrations due to fiber breakages	85
3.3.1	Static case	85
3.3.1.1	Single filament failure	85
3.3.1.2	Multi-filament failure	88
3.3.2	Dynamic case	94
3.4	Statistical tensile strength theories	98
3.4.1	Preliminary	98
3.4.2	Strength of individual fibers	102
3.4.3	Strength of fiber bundles	104
3.4.4	Correlations between single fiber and fiber bundle strengths	106
3.4.4.1	Analysis	106
3.4.4.2	Single fiber strength distribution	108
3.4.5	Experimental measurements of Weibull shape parameter	109
3.4.5.1	Single fiber tests	110
3.4.5.2	Loose bundle tests	113
3.4.6	Strength of unidirectional fiber composites	115
3.4.6.1	Equal load sharing	115
3.4.6.2	Idealized local load sharing	118
3.4.6.3	Monte-Carlo simulation	132
3.4.7	Strength of cross-ply composites	134
3.4.7.1	Energy absorption during multiple fracture	134
3.4.7.2	Transverse cracking of cross-ply laminates	136
3.4.7.3	Statistical analysis	145
3.4.7.4	Transverse cracking and Monte-Carlo simulation	150
3.4.8	Delamination in laminates of multi-directional plies	157

*Contents* xi

3.4.8.1	Free-edge delamination	160
3.4.8.2	General delamination problems	165
3.4.9	Enhancement of composite strength through fiber prestressing	166
<b>4</b>	<b>Short-fiber composites</b>	<b>169</b>
4.1	Introduction	169
4.2	Load transfer	169
4.2.1	A single short fiber	170
4.2.2	Fiber–fiber interactions	171
4.3	Elastic properties	176
4.3.1	Unidirectionally aligned short-fiber composites	177
4.3.1.1	Shear-lag analysis	177
4.3.1.2	Self-consistent method	177
4.3.1.3	Bound approach	178
4.3.1.4	Halpin–Tsai equation	181
4.3.2	Partially aligned short-fiber composites	182
4.3.3	Random short-fiber composites	187
4.4	Physical properties	194
4.4.1	Thermal conductivity	194
4.4.2	Thermoelastic constants	198
4.5	Viscoelastic properties	200
4.6	Strength	201
4.6.1	Unidirectionally aligned short-fiber composites	202
4.6.1.1	Fiber length considerations	203
4.6.1.2	Probabilistic strength theory	205
4.6.2	Partially oriented short-fiber composites	216
4.6.3	Random short-fiber composites	224
4.7	Fracture behavior	227
<b>5</b>	<b>Hybrid composites</b>	<b>231</b>
5.1	Introduction	231
5.2	Stress concentrations	233
5.2.1	Static case	233
5.2.2	Dynamic case	243
5.3	Tensile stress–strain behavior	247
5.3.1	Elastic behavior	249
5.3.2	First cracking strain	251
5.3.3	Differential Poisson’s effect	254
5.3.4	Differential thermal expansion	256

xii *Contents*

5.4	Strength theories	256
5.4.1	Rule-of-mixtures	257
5.4.2	Probabilistic initial failure strength	257
5.4.3	Probabilistic ultimate failure strength	262
5.5	Softening strips	273
5.6	Mechanical properties	275
5.7	Property optimization analysis	279
5.7.1	Constitutive relations	279
5.7.2	Graphical illustration of performance optimization	282
<b>6</b>	<b>Two-dimensional textile structural composites</b>	<b>285</b>
6.1	Introduction	285
6.2	Textile preforms	287
6.2.1	Wovens	288
6.2.2	Knits	292
6.2.3	Braids	294
6.3	Methodology of analysis	300
6.4	Mosaic model	302
6.5	Crimp (fiber undulation) model	308
6.6	Bridging model and experimental confirmation	314
6.7	Analysis of the knee behavior and summary of stiffness and strength modeling	319
6.8	In-plane thermal expansion and thermal bending coefficients	327
6.9	Hybrid fabric composites: mosaic model	335
6.9.1	Definitions and idealizations	336
6.9.2	Bounds of stiffness and compliance constants	340
6.9.2.1	Iso-strain	341
6.9.2.2	Iso-stress	343
6.9.3	One-dimensional approximation	344
6.9.4	Numerical results	345
6.10	Hybrid fabric composites: crimp and bridging models	348
6.10.1	Crimp model	349
6.10.2	Bridging model	352
6.10.3	Numerical results and summary of thermoelastic properties	354
6.11	Triaxial woven fabric composites	356
6.11.1	Geometrical characteristics	356
6.11.2	Analysis of thermoelastic behavior	358
6.11.3	Biaxial non-orthogonal woven fabric composites	365

*Contents* xiii

6.12	Nonlinear stress-strain behavior	366
6.13	Mechanical properties	368
6.13.1	Friction and wear behavior	368
6.13.2	Notched strength	371
<b>7</b>	<b>Three-dimensional textile structural composites</b>	<b>374</b>
7.1	Introduction	374
7.2	Processing of textile preforms	376
7.2.1	Braiding	377
7.2.1.1	2-step braiding	377
7.2.1.2	4-step braiding	379
7.2.1.3	Solid braiding	382
7.2.2	Weaving	382
7.2.2.1	Angle-interlock multi-layer weaving	383
7.2.2.2	Orthogonal weaving	387
7.2.3	Stitching	387
7.2.4	Knitting	389
7.3	Processing windows for 2-step braids	389
7.3.1	Packing of fibers and yarn cross-sections	390
7.3.2	Unit cell of the preform	395
7.3.3	Criterion for yarn jamming	398
7.4	Yarn packing in 4-step braids	402
7.4.1	Unit cell of the preform	402
7.4.2	Criterion for yarn jamming	403
7.5	Analysis of thermoelastic behavior of composites	405
7.5.1	Elastic strain-energy approach	406
7.5.2	Fiber inclination model	407
7.5.3	Macro-cell approach	414
7.5.3.1	Geometric relations	414
7.5.3.2	Elastic constants	416
7.6	Structure-performance maps of composites	419
7.7	Mechanical properties of composites	428
7.7.1	Tensile and compressive behavior	428
7.7.2	Shear behavior	431
7.7.3	Fracture behavior	432
7.7.3.1	In-plane fracture	432
7.7.3.2	Interlaminar fracture	435
7.7.4	Impact	440
<b>8</b>	<b>Flexible composites</b>	<b>443</b>
8.1	Introduction	443
8.2	Cord/rubber composites	445

xiv *Contents*

8.2.1	Rubber and cord properties	446
8.2.2	Unidirectional composites	447
8.2.3	Laminated composites	449
8.2.4	Cord loads in tires	453
8.3	Coated fabrics	456
8.4	Nonlinear elastic behavior – incremental analysis	459
8.4.1	Geometry of wavy fibers	460
8.4.2	Axial tensile behavior	462
8.4.2.1	Iso-phase model	462
8.4.2.2	Random-phase model	465
8.4.2.3	Nonlinear tensile stress-strain behavior	467
8.4.3	Transverse tensile behavior	471
8.4.3.1	Iso-phase model	471
8.4.3.2	Random-phase model	472
<b>9</b>	<b>Nonlinear elastic finite deformation of flexible composites</b>	<b>474</b>
9.1	Introduction	474
9.2	Background	478
9.2.1	Tensor notation	478
9.2.2	Lagrangian and Eulerian descriptions	480
9.3	Constitutive relations based on the Lagrangian description	485
9.3.1	Finite deformation of a composite lamina	485
9.3.2	Constitutive equations for a composite lamina	487
9.3.2.1	Strain-energy function	487
9.3.2.2	General constitutive equations for a unidirectional lamina	488
9.3.2.3	Pure homogeneous deformation	490
9.3.2.4	Simple shear	492
9.3.2.5	Simple shear superposed on simple extension	495
9.3.3	Constitutive equations of flexible composite laminates	499
9.3.3.1	Constitutive equations	499
9.3.3.2	Homogeneous deformation	500
9.3.3.3	Simple extension of a symmetric composite laminate	502
9.3.4	Determination of elastic constants	505
9.3.4.1	Tensile properties	505
9.3.4.2	Shear properties	506

*Contents*

xv

9.4	Constitutive relations based on the Eulerian description	508
9.4.1	Stress-energy function	509
9.4.2	General constitutive equations	511
9.4.3	Pure homogeneous deformation	514
9.4.4	Simple shear superposed on simple extension	515
9.4.5	Determination of elastic compliance constants	517
9.5	Elastic behavior of flexible composites reinforced with wavy fibers	519
9.5.1	Introduction	519
9.5.2	Longitudinal elastic behavior based on the Lagrangian approach	520
9.5.3	Longitudinal elastic behavior based on the Eulerian approach	522
	<b>References</b>	<b>526</b>
	<b>Author index</b>	<b>556</b>
	<b>Subject index</b>	<b>563</b>