CAMBRIDGE STUDIES IN ADVANCED MATHEMATICS 20

Introductory lectures on Siegel modular forms

Already published

1 W.M.L. Holcombe *Algebraic automata theory*
2 K. Petersen *Ergodic theory*
3 P.T. Johnstone *Stone spaces*
4 W.H. Schikhof *Ultrametric calculus*
5 J-P. Kahane *Some random series of functions*, second edition
6 H. Cohn *Introduction to the construction of class fields*
7 J. Lambek & P.J. Scott *Introduction to higher-order categorical logic*
8 H. Matsumura *Commutative ring theory*
9 C.B. Thomas *Characteristic classes and the cohomology of finite groups*
10 M. Aschbacher *Finite group theory*
11 J.L. Alperin *Local representation theory*
12 P. Koosis *The logarithmic integral: 1*
13 A. Pietsch *Eigenvalues and s-numbers*
14 S.J. Patterson *Introduction to the theory of the Riemann zeta-function*
15 H-J. Baues *Algebraic homotopy*
16 V.S. Varadarajan *Introduction to harmonic analysis on semisimple Lie groups*
17 W. Dicks & M.J. Dunwoody *Groups acting on graphs*
18 L.J. Corwin & F. P. Greenleaf *Representations of nilpotent Lie groups and their applications*
20 H. Klingen *Introduction to modular functions*
22 M. Collins *Representations and characters of finite groups*

Introductory lectures on Siegel modular forms

HELMUT KLINGEN

Professor of Mathematics
University of Freiburg

CAMBRIDGE UNIVERSITY PRESS

Cambridge

New York Port Chester Melbourne Sydney

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First published 1990

British Library cataloguing in publication data
Klingen, Helmut
Introductory lectures on Siegel modular forms.
1. Mathematics. Automorphic functions
I. Title
515.7

Library of Congress cataloguing in publication data
Klingen, Helmut.
Introductory lectures on Siegel modular forms/Helmut Klingen.
p. cm. – (Cambridge studies in advanced mathematics; 20)
Bibliography: p.
Includes index.
ISBN 0 521 35052 2
1. Siegel domains. 2. Modular groups. I. Title. II. Series.
QA331.K615 1990 515–dc20 89-9740 CIP

ISBN 0 521 35052 2

Transferred to digital printing 2003

AO

To my family,
Anita, Christoph and Philipp

Contents

Preface

The theory of automorphic functions in one complex variable was created during the second half of the nineteenth and the beginning of the twentieth centuries. Important contributions are due to such illustrious mathematicians as F. Klein, P. Koebe and H. Poincaré. Two sources may be traced: the uniformization theory of algebraic functions, and certain topics in number theory. Automorphic functions with respect to groups with compact quotient space on the one hand and elliptic modular functions on the other are examples of these two aspects. In several complex variables there is no analogue of uniformization theory; the class of automorphic functions which can be considered becomes much narrower, and the underlying groups are, in general, arithmetically defined.

In the mid-1930s C.L. Siegel discovered a new type of automorphic forms and functions in connection with his famous investigations on the analytic theory of quadratic forms. He denoted these functions as 'modular functions of degree *n*'; nowadays they are called 'Siegel modular functions'. Next to Abelian functions they are the most important example of automorphic functions in several complex variables, and they very soon became a touchstone to test the efficiency of general methods in several complex variables and other fields. Only recently, Hilbert modular functions have achieved a similar position due to the progress made in that area by K. Doi, F. Hirzebruch, F.W. Knöller, H. Naganuma and D. Zagier, amongst others. Siegel himself developed many powerful methods, and a steadily growing group of mathematicians increased the knowledge of Siegel modular forms, or found similar types of functions such as Hermitian modular forms, Hilbert–Siegel modular forms, or recently modular forms on half-spaces of quaternions. The need for a unified comprehensive treatment of these different but related theories became obvious. This was realized within the framework of arithmetically defined subgroups of algebraic groups and corresponding automorphic functions by the impressive work of W.L. Baily, A. Borel, R.P. Langlands and I.I. Pjateckij-Šapiro around 1965. Since then, further progress has been achieved only partly in this generality, but also in much more concrete situations, for instance A.N. Andrianov's results on Hecke's theory or the work of E. Freitag, D. Mumford and Y.-S. Tai on the structure of the function fields.

This book consists of lectures on Siegel modular forms that I have delivered in steadily improved versions, first in 1968 at the Tata Institute of Fundamental Research in Bombay, and afterwards on several occasions at the University of Freiburg. The audience in mind was composed of students who had taken only a one-complex-variable course besides having some basic knowledge in algebra, number theory and topology. In particular, in order to understand this book, the reader needs no knowledge of several complex variables, except perhaps the concept of a holomorphic or meromorphic function. The lectures were designed for a one-semester course with the intention of offering an easily accessible partial survey of the elementary parts of an exceptionally active field in mathematics. Consequently the selection of topics can by no means claim to be complete; even essential subjects like Satake's compactification or Hecke's theory cannot be included in this book. We restrict ourselves to the full modular group neglecting technical difficulties arising from subgroups. The reader, however, should feel encouraged to deal with the more advanced parts of the theory afterwards, using other books or the original literature recommended in the text.

Formulas, theorems etc. are numbered separately in each section. If a reference to a previous section is cited, the number of the section is placed in front of the number of the formula, theorem etc. For example, (4.1) means formula (1) of §4.

Acknowledgements are due to Siegfried Böcherer and Petra Ploch for their valuable comments and detailed reading of the manuscript. I am much indebted to Ruth Müller for her careful typing of the manuscript and her patience. Finally, I would like to take the opportunity to thank Cambridge University Press for the invitation and encouragement to write this book.

Freiburg, 1988 H. Klingen